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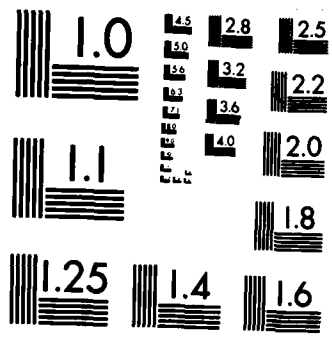
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TABLES FOR DISTRIBUTIONS OF QUADRATIC FORMS

BY

MARY ELLEN BOCK and HERBERT SOLOMON

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For the Office of Naval Research

Herbert Solomon, Project Director

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DEPARTMENT OF STATISTICS  
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# Tables For Distributions Of Quadratic Forms

by

Mary Ellen Bock and Herbert Solomon

## 1. Introduction.

The tables given here provide probabilities of the form

$$P\left[\sum_{i=1}^n a_i {}^{(i)}\chi_1^2 \leq t\right]$$

where  $a_i > 0$ ,  $\sum_{i=1}^n a_i = 1$  and  $a_i {}^{(i)}\chi_1^2$  variables with one degree of freedom, for  $n=2,3$ .

The probabilities are needed for a variety of problems. Solomon (1960) illustrates their use in hitting probabilities. Moore and Spruill (1975) survey their use for the asymptotic distribution of certain chi-square goodness-of-fit tests. They also arise as the distribution of the trace of a Wishart matrix as noted by Pillai (1973). Bock (1984) discusses other applications.

Section 2 provides formulas for the missing table values as well as formulas that reduce probabilities of the form

$$P\left[\sum_{i=1}^n a_i {}^{(i)}\chi_1^2 < t\right]$$

for  $n > 3$  to expressions involving probabilities found in this table (or in tables of Johnson and Kotz (1967, 1968)).

Some of the values from these tables are given in Owen (1962). Harter (1960) gives a table for a linear combination of two independent chi-square variables. Marsaglia (1960) gives tables for linear combinations of two or three independent chi-square variables each with one degree of freedom.

The extensive tables for  $n = 2, 3$  given in this report were given in an unpublished Technical Report, Solomon (1960). A number of requests over the years plus the additional uses for it discussed in this paper and by Bock (1984) suggest that its republication is merited.

## 2. Formulas for Missing Values.

The following formulas enable one to fill in some of the missing values in the tables.

An unpublished result of David Kleinecke (see Ruben (1962)) proves that for  $0 < p < 1$ ,

$$(1) \quad P[p^{(1)} \chi_1^2 + (1-p)^{(2)} \chi_1^2 < t] \\ = P[\chi_{2,A}^2 < B] - P[\chi_{2,B}^2 < A]$$

where

$$A = \frac{t}{4} (p^{-1/2} - (1-p)^{-1/2})^2$$

and

$$B = \frac{t}{4} (p^{-1/2} + (1-p)^{-1/2})^2$$



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where  $(j) \chi_1^2$  are independent chi-square variables each with one degree of freedom for  $j=1$  and  $2$  and  $\chi_{2,\lambda}^2$  is a noncentral chi-square random variable with two degrees of freedom and noncentrality parameter  $\lambda$ .

(Thus for  $X \sim N(\theta, I_2)$ , we have

$$\|x\|^2 = x_1^2 + x_2^2 \sim \chi_{2,\lambda}^2$$

with

$$\lambda = \|\theta\|^2 = \theta_1^2 + \theta_2^2 .)$$

Haynam, Govindarajulu, Leone and Siefert (1982,1983) give extensive tables for noncentral chi-square distributions.

The distribution of a linear combination of three independent chi-square random variables each with one degree of freedom is simplified if exactly two of the three coefficients are equal by the results of Bock (1984). For  $0 < p < 1/3$ ,

$$\begin{aligned} (2) \quad & P[p^{(1)} \chi_1^2 + p^{(2)} \chi_1^2 + (1-2p)^{(3)} \chi_1^2 > t] \\ & = P[\chi_1^2 > t/(1-2p)] + e^{-t/2(1-2p)} \left( \frac{4p}{\pi(1-3p)} \right)^{1/2} \mathcal{D}(x) \end{aligned}$$

where

$$x = \left[ \frac{t}{2} (p^{-1} - (1-2p)^{-1}) \right]^{1/2}$$

and  $\mathcal{D}(y)$  is Dawson's function tabled in Abramowitz and Stegun (1964), i.e.

$$D(y) = e^{-y^2} \int_0^y e^{u^2} du .$$

For  $\frac{1}{3} < p < \frac{1}{2}$ ,

$$\begin{aligned} (3) \quad & P[p^{(1)}\chi_1^2 + p^{(2)}\chi_1^2 + (1-2p)^{(3)}\chi_1^2 < t] \\ & = P[\chi_1^2 > t/(1-2p)] + e^{-t/2p} \left(\frac{p}{3p-1}\right)^{1/2} P[\chi_1^2 < t((1-2p)^{-1} - p^{-1})] \end{aligned}$$

The values in these tables may also be used to fill in certain missing values in the tables of Johnson and Kotz (1967,1968) for the cumulative distribution functions of linear combination of four or five independent chi-square random variables each with one degree of freedom. In particular, when several of the coefficients in the linear combination are the same, the following formulas from Bock (1984) apply.

For a linear combination of size  $n \geq 3$  independent chi-square variables, when the two largest coefficients  $c_0$  are equal

(i.e.  $c_0 > \max_{1 \leq i \leq n-2} \{c_i\}$ ) we have

$$\begin{aligned} (4) \quad & P\left[\sum_{i=1}^{n-2} c_i^{(1)}\chi_1^2 + c_0^{(n-1)}\chi_1^2 + c_0^{(n)}\chi_1^2 < t\right] \\ & = P\left[\sum_{i=1}^{n-2} c_i^{(1)}\chi_1^2 < t\right] - \alpha P\left[\sum_{i=1}^{n-2} c_i^{*(1)}\chi_1^2 < t\right] \end{aligned}$$

where

$$c_i^* = (c_i^{-1} - c_0^{-1})^{-1}$$



and

$$\alpha = e^{-t/2c_0} \prod_{i=1}^{n-2} \left( \frac{c_i^*}{c_i} \right)^{1/2}.$$

Thus if  $n=4$  or  $5$ , values in the Solomon table may be used to fill in missing values in the Johnson and Kotz tables. (If  $n=6$  or  $7$ , the formula (4) enables one to use the Johnson and Kotz tables to provide the missing probabilities.)

For a linear combination of five independent chi-square variables when  $c_1 = c_2 = c_3$  and  $c_4 = c_5$ , the formula may be written as

$$\begin{aligned} (5) \quad & P[c_1^{(1)} \chi_1^2 + c_1^{(2)} \chi_1^2 + c_1^{(3)} \chi_1^2 + c_5^{(4)} \chi_1^2 + c_5^{(5)} \chi_1^2 < t] \\ &= \frac{c_1}{(c_1 - c_5)} P[\chi_3^2 < \frac{t}{c_1}] \\ &+ \frac{c_5}{(c_5 - c_1)} P[c_1^{(1)} \chi_1^2 + c_5^{(2)} \chi_1^2 + c_5^{(3)} \chi_1^2 < t] \end{aligned}$$

If in a linear combination of five independent chi-square variables there are exactly three distinct coefficients so that  $c_1 = c_2$ ,  $c_3 = c_4$ , it can be shown that

$$\begin{aligned} (6) \quad & P[c_1^{(1)} \chi_1^2 + c_1^{(2)} \chi_1^2 + c_3^{(3)} \chi_1^2 + c_3^{(4)} \chi_1^2 + c_5^{(5)} \chi_1^2 > t] \\ &= \frac{c_1^2}{(c_1 - c_5)(c_1 - c_3)} P[c_1^{(1)} \chi_1^2 + c_1^{(2)} \chi_1^2 + c_5^{(3)} \chi_1^2 > t] \\ &+ \frac{c_3^2}{(c_3 - c_5)(c_3 - c_1)} P[c_3^{(1)} \chi_1^2 + c_3^{(2)} \chi_1^2 + c_5^{(3)} \chi_1^2 > t]. \end{aligned}$$

### 3. Examples.

Example 1. This example shows how one might fill in two missing values in these tables for a linear combination of three independent chi-square random variables when two of the coefficients are alike.

Using equation (3),

$$P[.45^{(1)}\chi_1^2 + .45^{(2)}\chi_1^2 + .1^{(3)}\chi_1^2 < .75]$$

$$= P[\chi_1^2 > 7.5] + .4928P[\chi_1^2 < 5.833]$$

$$= .4928.$$

Note that in the tables  $P[.4^{(1)}\chi_1^2 + .4^{(2)}\chi_1^2 + .2^{(3)}\chi_1^2 < .75] = .4880$ .

Using equation (2),

$$P[.25^{(1)}\chi_1^2 + .25^{(2)}\chi_1^2 + .5^{(3)}\chi_1^2 > 1.19]$$

$$= P[\chi_1^2 > 2.38] + .3433D(1.09)$$

$$= .3044, \text{ or}$$

$$P[.25^{(1)}\chi_1^2 + .25^{(2)}\chi_1^2 + .5^{(3)}\chi_1^2 < 1.19] = .6956 .$$

Note that in the Tables

$$P[.1^{(1)}\chi_1^2 + .4^{(2)}\chi_1^2 + .5^{(3)}\chi_1^2 < 1.19] = .6987 .$$

Example 2. This example shows how the tables in this report might be used to fill in a probability that is not in the Johnson and Kotz tables, using equation (4). (It shows that the formula is robust for substitution of approximate values of the probabilities in these tables.) The following probability

$$(*) = P[1.8^{(1)}\chi_1^2 + 1.8^{(2)}\chi_1^2 + .6^{(3)}\chi_1^2 + .5^{(4)}\chi_1^2 + .3^{(5)}\chi_1^2 < 9.930]$$

is tabled in Johnson and Kotz (1968) as .9.

According to equation (4), this may be written as

$$(*) = P[.429^{(1)}\chi_1^2 + .357^{(2)}\chi_1^2 + .214^{(3)}\chi_1^2 < 7.09] \\ - (.10)P[.461^{(1)}\chi_1^2 + .355^{(2)}\chi_1^2 + .184^{(3)}\chi_1^2 < 5.08] .$$

The particular coefficients given for the linear combinations of three independent chi-square variables do not appear in the tables of Solomon. However, rough approximations such as substituting

$$P[.4^{(1)}\chi_1^2 + .4^{(2)}\chi_1^2 + .2^{(3)}\chi_1^2 < 7]$$

for the first probability and

$$P[.5^{(1)}\chi_1^2 + .3^{(2)}\chi_1^2 + .2^{(3)}\chi_1^2 < 5]$$

for the second probability in the last representation for (\*) give the value .9001.

The value .5 is given in Johnson and Kotz (1968) for

$$\begin{aligned}
 (***) &= P[1.8^{(1)}\chi_1^2 + 1.8^{(2)}\chi_1^2 + .6^{(3)}\chi_1^2 + \\
 &\quad .5^{(4)}\chi_1^2 + .3^{(5)}\chi_1^2 < 4.016] .
 \end{aligned}$$

By equation (4), this is

$$\begin{aligned}
 (***) &= P[.429^{(1)}\chi_1^2 + .357^{(2)}\chi_1^2 + .214^{(3)}\chi_1^2 < 2.8686] \\
 &\quad - (.5165)P[.461^{(1)}\chi_1^2 + .355^{(2)}\chi_1^2 + .184^{(3)}\chi_1^2 < 2.0562] .
 \end{aligned}$$

Substituting  $P[.4^{(1)}\chi_1^2 + .4^{(2)}\chi_1^2 + .2^{(3)}\chi_1^2 < 2.85]$  for the first probability in (\*\*\*) and  $P[.5^{(1)}\chi_1^2 + .3^{(2)}\chi_1^2 + .2^{(3)}\chi_1^2 < 2.05]$  for the second probability in (\*\*\*), we have .4998 as an estimate of (\*\*\*) .

For the probability

$$\begin{aligned}
 (**) &= P[1.8^{(1)}\chi_1^2 + 1.8^{(2)}\chi_1^2 + .6^{(3)}\chi_1^2 + \\
 &\quad + .5^{(4)}\chi_1^2 + .3^{(5)}\chi_1^2 < .936]
 \end{aligned}$$

tabled in Johnson and Kotz as .05, we have

$$\begin{aligned}
 (**) &= P[.429^{(1)}\chi_1^2 + .357^{(2)}\chi_1^2 + .214^{(3)}\chi_1^2 < .6683] \\
 &\quad - (1.2152)P[.461^{(1)}\chi_1^2 + .355^{(2)}\chi_1^2 + .184^{(3)}\chi_1^2 < .4788] .
 \end{aligned}$$

Substituting  $P[.4 \chi_1^{(1)2} + .4 \chi_1^{(1)2} + .2 \chi_1^{(3)2} < .67]$  for the first probability in (\*\*) and  $P[.5 \chi_1^{(1)2} + .3 \chi_1^{(2)2} + .2 \chi_1^{(3)2} < .48]$  for the second probability in (\*\*), we have .0510 as the estimate of (\*\*).

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Table 1  
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

| $a_2, a_1$ |           |          |          |          |          |
|------------|-----------|----------|----------|----------|----------|
| .8,.2      | .875,.125 | .9,.1    | .95,.05  | .99,.01  | 1,0      |
| 93149416   | 92433215  | 92179771 | 91663484 | 91246447 | 91142042 |
| 93279256   | 92564658  | 92312098 | 91797755 | 91382308 | 91278275 |
| 93406451   | 92693645  | 92442015 | 91929691 | 91515852 | 91412242 |
| 93531063   | 92820231  | 92569577 | 92059340 | 91647194 | 91543986 |
| 93653149   | 92944464  | 92694825 | 92186744 | 91776330 | 91673546 |
|            |           |          |          |          |          |
| 93772767   | 93066393  | 92817810 | 92311955 | 91903321 | 91800966 |
| 93889971   | 93186067  | 92938577 | 92435009 | 92028198 | 91926284 |
| 94074816   | 93303532  | 93057168 | 92555948 | 92150985 | 92049539 |
| 94117355   | 93418830  | 93173630 | 92674807 | 92271765 | 92170769 |
| 94227637   | 93532008  | 93288002 | 92791637 | 92390515 | 92290011 |
|            |           |          |          |          |          |
| 94335713   | 93643110  | 93400327 | 92906472 | 92507333 | 92407302 |
| 94441633   | 93752176  | 93510646 | 93019346 | 92622204 | 92522677 |
| 94545444   | 93859247  | 93618998 | 93130304 | 92735231 | 92636171 |
| 94647190   | 93964363  | 93725420 | 93239376 | 92846345 | 92747818 |
| 94746918   | 94067566  | 93829953 | 93346595 | 92955674 | 92857652 |
|            |           |          |          |          |          |
| 94844671   | 94168892  | 93932631 | 93452001 | 93063201 | 92965705 |
| 94940492   | 94268378  | 94033492 | 93555627 | 93168994 | 93072009 |
| 95034424   | 94366062  | 94132570 | 93657506 | 93273033 | 93176596 |
| 95126506   | 94461979  | 94229903 | 93757669 | 93375405 | 93279496 |
| 95216779   | 94556164  | 94325520 | 93856147 | 93476114 | 93380739 |
|            |           |          |          |          |          |
| 95305281   | 94648651  | 94419454 | 93952974 | 93575191 | 93480355 |
| 95392049   | 94739474  | 94511742 | 94048175 | 93672638 | 93578373 |
| 95477123   | 94828665  | 94602412 | 94141789 | 93768529 | 93674820 |
| 95560535   | 94916256  | 94691496 | 94233836 | 93862880 | 93769725 |
| 95642323   | 95002279  | 94779023 | 94324350 | 93955710 | 93863114 |
|            |           |          |          |          |          |
| 95801161   | 95169740  | 94949527 | 94500883 | 94136908 | 94045452 |
| 95953903   | 95331284  | 95114154 | 94671599 | 94312352 | 94222040 |
| 96100799   | 95487137  | 95273124 | 94836721 | 94482216 | 94393076 |
| 96242091   | 95637517  | 95426645 | 94996435 | 94646726 | 94558750 |
| 96378010   | 95782629  | 95574919 | 95150943 | 94806052 | 94719246 |
|            |           |          |          |          |          |
| 96508773   | 95922672  | 95718140 | 95300421 | 94960359 | 94874739 |
| 96634592   | 96057837  | 95856490 | 95445047 | 95109840 | 95025398 |
| 96755667   | 96188305  | 95990151 | 95584992 | 95254633 | 95171386 |
| 96872188   | 96314252  | 96119295 | 95720418 | 95394928 | 95312860 |
| 96984338   | 96435845  | 96244081 | 95851484 | 95530832 | 95449970 |
|            |           |          |          |          |          |
| 97092291   | 96553245  | 96364670 | 95978335 | 95662570 | 95582862 |
| 97196216   | 96666606  | 96481210 | 96101125 | 95790169 | 95711675 |
| 97296272   | 96776076  | 96593846 | 96219988 | 95913869 | 95836544 |
| 97392611   | 96881796  | 96702719 | 96335059 | 96033776 | 95957598 |
| 97485380   | 96983904  | 96807962 | 96446468 | 96149937 | 96074963 |

Table 1  
 $P[a_1^{(1)}x_1^2 + a_2^{(2)}x_1^2 \leq t]$

| $a_2, a_1$ |          |          |                            |          |          |
|------------|----------|----------|----------------------------|----------|----------|
| t          | .5, .5   | .6, .4   | $\frac{2}{3}, \frac{1}{3}$ | .7, .3   | .75, .25 |
| 2.90       | 94497680 | 94352595 | 94088753                   | 93904350 | 93562601 |
| 2.925      | 94633530 | 94486970 | 94221035                   | 94035584 | 93692682 |
| 2.95       | 94766030 | 94618090 | 94350229                   | 94163834 | 93819951 |
| 2.975      | 94895260 | 94746036 | 94476410                   | 94289174 | 93944476 |
| 3.00       | 95021290 | 94870885 | 94599652                   | 94411674 | 94066323 |
| 3.025      | 95144220 | 94992713 | 94720026                   | 94531403 | 94185554 |
| 3.05       | 95264110 | 95111595 | 94837603                   | 94648428 | 94302229 |
| 3.075      | 95381040 | 95227605 | 94952450                   | 94762815 | 94416410 |
| 3.10       | 95495080 | 95340811 | 95064635                   | 94874626 | 94528155 |
| 3.125      | 95606310 | 95451284 | 95174222                   | 94983923 | 94637519 |
| 3.15       | 95714790 | 95559092 | 95281277                   | 95090767 | 94744560 |
| 3.175      | 95820290 | 95664298 | 95385856                   | 95195216 | 94849329 |
| 3.20       | 95923780 | 95766969 | 95488022                   | 95297328 | 94951880 |
| 3.225      | 96024420 | 95867163 | 95587835                   | 95397160 | 95052265 |
| 3.25       | 96122580 | 95964946 | 95685348                   | 95494765 | 95150532 |
| 3.275      | 96218310 | 96060373 | 95780621                   | 95590194 | 95246730 |
| 3.30       | 96311680 | 96153504 | 95873704                   | 95683501 | 95340907 |
| 3.325      | 96402750 | 96244395 | 95964652                   | 95774736 | 95433108 |
| 3.35       | 96491560 | 96333100 | 96053515                   | 95863947 | 95523378 |
| 3.375      | 96578190 | 96419673 | 96140344                   | 95951183 | 95611762 |
| 3.40       | 96662670 | 96504168 | 96225189                   | 96036489 | 95698302 |
| 3.425      | 96745070 | 96586632 | 96308095                   | 96119912 | 95783038 |
| 3.45       | 96825440 | 96667118 | 96389110                   | 96201495 | 95866014 |
| 3.475      | 96903830 | 96745673 | 96468277                   | 96281281 | 95947266 |
| 3.50       | 96980260 | 96822344 | 96545643                   | 96359313 | 96026834 |
| 3.55       | 97127540 | 96970220 | 96695139                   | 96510276 | 96181070 |
| 3.60       | 97267630 | 97111098 | 96837926                   | 96654698 | 96329009 |
| 3.65       | 97400890 | 97245316 | 96974319                   | 96792880 | 96470927 |
| 3.70       | 97527650 | 97373195 | 97104618                   | 96925106 | 96607089 |
| 3.75       | 97648230 | 97495039 | 97229107                   | 97051648 | 96737742 |
| 3.80       | 97762920 | 97611138 | 97348056                   | 97172763 | 96863126 |
| 3.85       | 97872030 | 97721769 | 97461721                   | 97288699 | 96983468 |
| 3.90       | 97975810 | 97827192 | 97570348                   | 97399686 | 97098983 |
| 3.95       | 98074530 | 97927658 | 97674169                   | 97505949 | 97209878 |
| 4.00       | 98168440 | 98023403 | 97773406                   | 97607699 | 97316349 |
| 4.05       | 98257760 | 98114654 | 97868269                   | 97705137 | 97418584 |
| 4.10       | 98342730 | 98201625 | 97958959                   | 97798455 | 97516761 |
| 4.15       | 98423560 | 98284521 | 98045668                   | 97887837 | 97611052 |
| 4.20       | 98500440 | 98363535 | 98128576                   | 97973457 | 97701618 |
| 4.25       | 98573580 | 98438852 | 98207857                   | 98055480 | 97788616 |



Table 1  
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

| $a_2, a_1$ |            |          |          |          |          |
|------------|------------|----------|----------|----------|----------|
| .8, .2     | .875, .125 | .9, .1   | .95, .05 | .99, .01 | 1, 0     |
| 86263825   | 85715512   | 85486633 | 84992446 | 84580226 | 84476086 |
| 86487318   | 85927381   | 85695932 | 85198058 | 84783662 | 84679036 |
| 86706725   | 86135706   | 85901829 | 85400485 | 84984077 | 84878984 |
| 86922137   | 86340555   | 86104390 | 85599797 | 85181463 | 85075983 |
| 87133636   | 86541999   | 86303684 | 85796069 | 85375952 | 85270087 |
|            |            |          |          |          |          |
| 87341308   | 86740105   | 86499768 | 85989320 | 85567584 | 85461346 |
| 87545233   | 86934944   | 86692708 | 86179617 | 85756377 | 85649811 |
| 87745491   | 87126572   | 86882566 | 86367028 | 85942409 | 85835531 |
| 87942156   | 87315059   | 87069397 | 86551594 | 86125695 | 86018554 |
| 88135309   | 87500467   | 87253256 | 86733368 | 86306329 | 86198926 |
|            |            |          |          |          |          |
| 88325019   | 87682853   | 87434203 | 86912408 | 86484302 | 86376694 |
| 88511360   | 87862277   | 87612299 | 87088751 | 86659725 | 86551902 |
| 88694400   | 88038792   | 87787583 | 87262453 | 86832593 | 86724594 |
| 88874208   | 88212460   | 87960120 | 87433549 | 87002924 | 86894813 |
| 89050849   | 88383329   | 88129951 | 87602100 | 87170834 | 87062600 |
|            |            |          |          |          |          |
| 89224389   | 88551453   | 88297131 | 87768138 | 87336360 | 87227997 |
| 89394886   | 88716891   | 88461708 | 87931716 | 87499465 | 87391044 |
| 89562410   | 88879684   | 88623732 | 88092884 | 87660302 | 87551780 |
| 89727016   | 89039889   | 88783243 | 88251664 | 87818801 | 87710244 |
| 89888763   | 89197547   | 88940289 | 88408108 | 87975068 | 87866474 |
|            |            |          |          |          |          |
| 90047710   | 89352710   | 89094918 | 88562252 | 88129100 | 88020506 |
| 90203910   | 89505421   | 89247168 | 88714136 | 88280942 | 88172376 |
| 90357419   | 89655726   | 89397085 | 88863803 | 88430684 | 88322120 |
| 90508291   | 89803669   | 89544707 | 89011300 | 88578273 | 88469773 |
| 90656574   | 89949296   | 89690075 | 89156641 | 88723804 | 88615368 |
|            |            |          |          |          |          |
| 90838368   | 90128131   | 89868684 | 89335384 | 88902873 | 88794520 |
| 91016302   | 90303487   | 90043908 | 89510840 | 89078777 | 88970573 |
| 91190461   | 90475443   | 90215821 | 89683169 | 89251645 | 89143588 |
| 91360939   | 90644073   | 90384498 | 89852399 | 89421536 | 89313627 |
| 91527827   | 90809451   | 90550009 | 90018595 | 89588445 | 89480748 |
|            |            |          |          |          |          |
| 91691206   | 90971652   | 90712419 | 90181825 | 89752501 | 89645009 |
| 91851164   | 91130742   | 90871795 | 90342149 | 89913734 | 89806466 |
| 92007780   | 91286789   | 91028199 | 90499623 | 90072174 | 89965174 |
| 92161132   | 91439862   | 91181701 | 90654310 | 90227944 | 90121186 |
| 92311297   | 91590019   | 91332356 | 90806259 | 90381007 | 90274555 |
|            |            |          |          |          |          |
| 92458351   | 91737328   | 91480227 | 90955527 | 90531486 | 90425332 |
| 92602368   | 91881848   | 91625366 | 91102168 | 90679403 | 90573567 |
| 92743413   | 92023642   | 91767837 | 91246231 | 90824821 | 90719308 |
| 92881560   | 92162762   | 91907692 | 91387777 | 90967772 | 90862604 |
| 93016873   | 92299268   | 92044984 | 91526846 | 91108291 | 91003500 |

Table 1  
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

| t     | $a_2, a_1$ |          |                            |          |          |
|-------|------------|----------|----------------------------|----------|----------|
|       | .5, .5     | .6, .4   | $\frac{2}{3}, \frac{1}{3}$ | .7, .3   | .75, .25 |
| 2.02  | 86734450   | 86723695 | 86677978                   | 86626884 | 86492425 |
| 2.04  | 86997130   | 86981182 | 86926618                   | 86869786 | 86725700 |
| 2.06  | 87254600   | 87233579 | 87170376                   | 87107964 | 86954537 |
| 2.08  | 87506980   | 87480989 | 87409358                   | 87341510 | 87179029 |
| 2.10  | 87754360   | 87723512 | 87643660                   | 87570526 | 87399274 |
| 2.12  | 87996840   | 87961246 | 87873377                   | 87795105 | 87615363 |
| 2.14  | 88234520   | 88194288 | 88098605                   | 88015343 | 87827377 |
| 2.16  | 88467490   | 88422732 | 88319438                   | 88231331 | 88035411 |
| 2.18  | 88695850   | 88646669 | 88535966                   | 88443153 | 88239547 |
| 2.20  | 88919680   | 88866194 | 88748280                   | 88650903 | 88439864 |
| 2.22  | 89139090   | 89081390 | 88956463                   | 88854657 | 88636443 |
| 2.24  | 89354150   | 89292350 | 89160601                   | 89054504 | 88829367 |
| 2.26  | 89564950   | 89499157 | 89360775                   | 89250525 | 89018708 |
| 2.28  | 89771580   | 89701891 | 89557070                   | 89442796 | 89204542 |
| 2.30  | 89974120   | 89900634 | 89749563                   | 89631395 | 89386940 |
| 2.32  | 90172640   | 90095473 | 89938332                   | 89816403 | 89565977 |
| 2.34  | 90367240   | 90286477 | 90123458                   | 89997886 | 89741720 |
| 2.36  | 90557980   | 90473733 | 90305007                   | 90175921 | 89914238 |
| 2.38  | 90744940   | 90657309 | 90483056                   | 90350579 | 90083595 |
| 2.40  | 90928200   | 90837279 | 90657675                   | 90521927 | 90249859 |
| 2.42  | 91107840   | 91013718 | 90828936                   | 90690033 | 90413090 |
| 2.44  | 91283910   | 91186695 | 90996904                   | 90854966 | 90573350 |
| 2.46  | 91456500   | 91356279 | 91161648                   | 91016787 | 90730703 |
| 2.48  | 91625680   | 91522538 | 91323235                   | 91175561 | 90885206 |
| 2.50  | 91791500   | 91685540 | 91481725                   | 91331349 | 91036915 |
| 2.525 | 91994170   | 91884808 | 91675579                   | 91521978 | 91222708 |
| 2.55  | 92191830   | 92079211 | 91864814                   | 91708150 | 91404333 |
| 2.575 | 92384620   | 92268872 | 92049545                   | 91889981 | 91581893 |
| 2.60  | 92572640   | 92453905 | 92229887                   | 92067577 | 91755488 |
| 2.625 | 92756020   | 92634430 | 92405945                   | 92241047 | 91925218 |
| 2.65  | 92934880   | 92810554 | 92577831                   | 92410490 | 92091178 |
| 2.675 | 93109320   | 92982389 | 92745647                   | 92576008 | 92253459 |
| 2.70  | 93279450   | 93150042 | 92909495                   | 92737700 | 92412154 |
| 2.725 | 93445380   | 93313614 | 93069473                   | 92895659 | 92567350 |
| 2.75  | 93607210   | 93473209 | 93225679                   | 93049979 | 92719129 |
| 2.775 | 93765050   | 93628926 | 93378205                   | 93200750 | 92867578 |
| 2.80  | 93918990   | 93780858 | 93527142                   | 93348060 | 93012777 |
| 2.825 | 94069130   | 93929101 | 93672580                   | 93491992 | 93154803 |
| 2.85  | 94215570   | 94073746 | 93814606                   | 93632631 | 93293733 |
| 2.875 | 94358390   | 94214882 | 93953303                   | 93770057 | 93429642 |

Table 1  
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

| $a_2, a_1$ |            |          |          |          |          |
|------------|------------|----------|----------|----------|----------|
| .8, .2     | .875, .125 | .9, .1   | .95, .05 | .99, .01 | 1, 0     |
| 72577279   | 73168331   | 73253695 | 73235696 | 73105707 | 73063934 |
| 73075221   | 73615771   | 73685168 | 73642192 | 73497766 | 73452894 |
| 73562397   | 74053851   | 74107917 | 74041022 | 73882819 | 73834890 |
| 74039094   | 74482855   | 74522185 | 74432401 | 74260923 | 74210096 |
| 74505588   | 74903034   | 74928205 | 74816517 | 74632263 | 74578677 |
|            |            |          |          |          |          |
| 74962150   | 75314642   | 75326211 | 75193540 | 74997100 | 74940794 |
| 75409032   | 75717909   | 75716426 | 75563677 | 75355461 | 75296602 |
| 75846490   | 76113067   | 76099049 | 75927070 | 75707585 | 75646251 |
| 76274769   | 76500341   | 76474294 | 76283912 | 76053633 | 75989884 |
| 76694102   | 76879943   | 76842349 | 76634349 | 76393673 | 76327642 |
|            |            |          |          |          |          |
| 77104715   | 77252068   | 77203398 | 76978531 | 76727919 | 76659659 |
| 77506833   | 77616928   | 77557633 | 77316620 | 77056431 | 76986065 |
| 77900670   | 77974698   | 77905220 | 77648744 | 77379418 | 77306987 |
| 78286433   | 78325571   | 78246321 | 77975052 | 77696901 | 77622547 |
| 78664324   | 78669724   | 78581104 | 78295679 | 78009078 | 77932863 |
|            |            |          |          |          |          |
| 79034538   | 79007314   | 78909728 | 78610758 | 78316109 | 78238050 |
| 79397262   | 79338524   | 79232332 | 78920400 | 78617964 | 78538219 |
| 79752687   | 79663496   | 79549070 | 79224736 | 78914822 | 78833477 |
| 80100986   | 79982392   | 79860081 | 79523891 | 79206894 | 79123930 |
| 80442335   | 80295361   | 80165492 | 79817965 | 79494142 | 79409678 |
|            |            |          |          |          |          |
| 80776898   | 80602537   | 80465454 | 80107084 | 79776760 | 79690820 |
| 81104844   | 80904072   | 80760069 | 80391346 | 80054756 | 79967452 |
| 81426325   | 81200088   | 81049471 | 80670859 | 80328371 | 80239666 |
| 81741500   | 81490724   | 81333782 | 80945724 | 80597480 | 80507553 |
| 82050512   | 81776096   | 81613116 | 81216024 | 80862312 | 80771201 |
|            |            |          |          |          |          |
| 82353512   | 82056333   | 81887576 | 81481891 | 81123044 | 81030694 |
| 82650640   | 82331546   | 82157280 | 81743375 | 81379519 | 81286116 |
| 82942031   | 82601853   | 82422323 | 82000599 | 81631981 | 81537547 |
| 83227819   | 82867366   | 82682815 | 82253637 | 81880532 | 81785066 |
| 83508132   | 83128195   | 82938851 | 82502565 | 82125145 | 82028749 |
|            |            |          |          |          |          |
| 83783097   | 83384433   | 83190526 | 82747487 | 82365985 | 82268671 |
| 84052835   | 83636194   | 83437934 | 82988470 | 82603113 | 82504904 |
| 84317462   | 83883563   | 83681163 | 83225593 | 82836540 | 82737519 |
| 84577101   | 84126647   | 83920305 | 83458933 | 83066374 | 82966584 |
| 84831858   | 84365529   | 84155436 | 83688567 | 83292676 | 83192167 |
|            |            |          |          |          |          |
| 85081844   | 84600301   | 84386654 | 83914564 | 83515571 | 83414333 |
| 85327165   | 84831057   | 84614024 | 84136998 | 83735052 | 83633146 |
| 85567925   | 85057870   | 84837632 | 84355935 | 83951167 | 83848668 |
| 85804224   | 85280835   | 85057556 | 84571455 | 84164050 | 84060959 |
| 86036160   | 85500020   | 85273865 | 84783601 | 84373736 | 84270079 |

Table 1  
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

| $a_2, a_1$ |          |          |                            |          |          |
|------------|----------|----------|----------------------------|----------|----------|
| $t$        | .5,.5    | .6,.4    | $\frac{2}{3}, \frac{1}{3}$ | .7,.3    | .75,.25  |
| 1.22       | 70476980 | 70754227 | 71226559                   | 71533067 | 72049656 |
| 1.24       | 71061580 | 71330414 | 71786907                   | 72081909 | 72576172 |
| 1.26       | 71634600 | 71895024 | 72335719                   | 72619279 | 73091453 |
| 1.28       | 72196270 | 72448288 | 72873248                   | 73145447 | 73595768 |
| 1.30       | 72746820 | 72990448 | 73399738                   | 73660662 | 74089384 |
| 1.32       | 73286470 | 73521723 | 73915430                   | 74165175 | 74572563 |
| 1.34       | 73815430 | 74042340 | 74420559                   | 74659224 | 75045559 |
| 1.36       | 74333920 | 74552520 | 74915348                   | 75143048 | 75508615 |
| 1.38       | 74842140 | 75052472 | 75400030                   | 75616875 | 75961975 |
| 1.40       | 75340300 | 75542401 | 75874817                   | 76080933 | 76405865 |
| 1.42       | 75828600 | 76022525 | 76339926                   | 76535437 | 76840514 |
| 1.44       | 76307220 | 76493034 | 76795564                   | 76980601 | 77266144 |
| 1.46       | 76776370 | 76954127 | 77241939                   | 77416635 | 77682968 |
| 1.48       | 77236230 | 77405996 | 77679246                   | 77843747 | 78091194 |
| 1.50       | 77686980 | 77848828 | 78107683                   | 78262129 | 78491026 |
| 1.52       | 78128810 | 78282810 | 78527436                   | 78671975 | 78882662 |
| 1.54       | 78561890 | 78708123 | 78938697                   | 79073476 | 79266289 |
| 1.56       | 78986390 | 79124940 | 79341643                   | 79466821 | 79642102 |
| 1.58       | 79402490 | 79533437 | 79736456                   | 79852185 | 80010277 |
| 1.60       | 79810350 | 79933783 | 80123308                   | 80229743 | 80370992 |
| 1.62       | 80210130 | 80326141 | 80502369                   | 80599669 | 80724423 |
| 1.64       | 80602000 | 80710677 | 80873804                   | 80962133 | 81070735 |
| 1.66       | 80986100 | 81087550 | 81237776                   | 81317297 | 81410090 |
| 1.68       | 81362600 | 81456915 | 81594443                   | 81665313 | 81742652 |
| 1.70       | 81731650 | 81818921 | 81943963                   | 82006348 | 82068572 |
| 1.72       | 82093380 | 82173728 | 82286486                   | 82340552 | 82388002 |
| 1.74       | 82447960 | 82521468 | 82622157                   | 82668070 | 82701092 |
| 1.76       | 82795510 | 82862294 | 82951125                   | 82989049 | 83007982 |
| 1.78       | 83136180 | 83196345 | 83273531                   | 83303630 | 83308812 |
| 1.80       | 83470110 | 83523757 | 83589514                   | 83611952 | 83603722 |
| 1.82       | 83797420 | 83844663 | 83899208                   | 83914151 | 83892841 |
| 1.84       | 84118260 | 84159198 | 84202743                   | 84210357 | 84176296 |
| 1.86       | 84432740 | 84467489 | 84500253                   | 84500699 | 84454218 |
| 1.88       | 84740990 | 84769663 | 84791862                   | 84785305 | 84726728 |
| 1.90       | 85043140 | 85065845 | 85077696                   | 85064296 | 84993947 |
| 1.92       | 85339300 | 85356155 | 85357872                   | 85337796 | 85255984 |
| 1.94       | 85629600 | 85640709 | 85632513                   | 85605914 | 85512963 |
| 1.96       | 85914160 | 85919630 | 85901732                   | 85868774 | 85764990 |
| 1.98       | 86193080 | 86193027 | 86165642                   | 86126481 | 86012173 |
| 2.00       | 86466470 | 86461011 | 86424355                   | 86379147 | 86254615 |

Table 1  
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

| $a_2, a_1$ |            |          |          |          |          |
|------------|------------|----------|----------|----------|----------|
| .8, .2     | .875, .125 | .9, .1   | .95, .05 | .99, .01 | 1, 0     |
| 55102125   | 57433026   | 58198335 | 59402056 | 59953979 | 60055616 |
| 55558374   | 57849613   | 58595843 | 59759395 | 60288544 | 60385606 |
| 56008901   | 58260451   | 58987820 | 60112108 | 60619062 | 60711678 |
| 56453791   | 58665659   | 59374400 | 60460296 | 60945684 | 61033909 |
| 56893125   | 59065349   | 59755675 | 60804068 | 61268437 | 61352374 |
|            |            |          |          |          |          |
| 57327002   | 59459636   | 60131783 | 61143512 | 61587340 | 61567145 |
| 57755490   | 59848613   | 60502819 | 61478735 | 61902579 | 61978292 |
| 58178686   | 60232390   | 60868894 | 61809796 | 62214194 | 62285883 |
| 58596660   | 60611074   | 61230108 | 62136802 | 62522178 | 62589984 |
| 59009495   | 60984754   | 61586565 | 62459837 | 62826666 | 62890660 |
|            |            |          |          |          |          |
| 59417270   | 61353535   | 61938367 | 62778975 | 63127699 | 63187972 |
| 59820057   | 61717500   | 62285592 | 63094292 | 63425382 | 63481980 |
| 60217935   | 62076750   | 62628347 | 63405862 | 63719716 | 63772744 |
| 60610982   | 62431363   | 62966711 | 63713759 | 64010765 | 64060320 |
| 60990262   | 62781431   | 63300780 | 64018055 | 64298636 | 64344763 |
|            |            |          |          |          |          |
| 61382849   | 63127047   | 63630636 | 64318819 | 64583375 | 64626128 |
| 61761819   | 63468277   | 63956352 | 64616121 | 64864979 | 64904465 |
| 62136228   | 63805214   | 64278021 | 64910004 | 65143545 | 65179829 |
| 62506156   | 64137927   | 64595719 | 65200559 | 65419122 | 65452266 |
| 62871662   | 64466503   | 64909512 | 65487830 | 65691777 | 65721826 |
|            |            |          |          |          |          |
| 63232815   | 64791008   | 65219476 | 65771882 | 65961515 | 65988555 |
| 63589680   | 65111514   | 65525687 | 66052762 | 66228373 | 66252500 |
| 63942315   | 65428101   | 65828208 | 66330523 | 66492526 | 66513705 |
| 64290785   | 65740829   | 66127114 | 66605238 | 66753859 | 66772213 |
| 64635150   | 66049773   | 66422458 | 66876951 | 67012465 | 67028067 |
|            |            |          |          |          |          |
| 64975468   | 66354991   | 66714319 | 67145708 | 67268424 | 67281309 |
| 65311803   | 66656554   | 67002750 | 67411556 | 67521744 | 67531979 |
| 65644209   | 66954522   | 67287822 | 67674546 | 67772541 | 67780116 |
| 65972741   | 67248956   | 67569585 | 67934731 | 68020732 | 68025759 |
| 66297459   | 67539921   | 67848092 | 68192149 | 68266400 | 68268946 |
|            |            |          |          |          |          |
| 66935659   | 68111662   | 68395592 | 68698878 | 68750506 | 68748096 |
| 67559241   | 68670195   | 68930747 | 69195060 | 69224888 | 69217850 |
| 68168613   | 69215956   | 69453970 | 69681036 | 69690054 | 69678479 |
| 68764179   | 69749361   | 69965662 | 70157111 | 70146079 | 70130242 |
| 69346319   | 70270810   | 70466188 | 70623584 | 70593364 | 70573387 |
|            |            |          |          |          |          |
| 69915406   | 70780665   | 70955902 | 71080728 | 71032144 | 71008152 |
| 70471804   | 71279303   | 71435146 | 71528829 | 71462574 | 71434765 |
| 71015860   | 71767061   | 71904246 | 71968141 | 71884896 | 71853444 |
| 71547907   | 72244282   | 72363508 | 72398886 | 72299423 | 72264399 |
| 72068275   | 72711270   | 72813234 | 72821341 | 72706277 | 72667831 |

Table 1  
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

| t    | $a_2, a_1$ |          |                            |          |          |
|------|------------|----------|----------------------------|----------|----------|
|      | .5, .5     | .6, .4   | $\frac{2}{3}, \frac{1}{3}$ | .7, .3   | .75, .25 |
| .71  | 50835580   | 51289400 | 52112827                   | 52689866 | 53769827 |
| .72  | 51324770   | 51776715 | 52595900                   | 53169254 | 54240555 |
| .73  | 51809100   | 52259063 | 53073809                   | 53643359 | 54705777 |
| .74  | 52288610   | 52736483 | 53546615                   | 54112232 | 55165555 |
| .75  | 52763340   | 53209034 | 54014369                   | 54575935 | 55619969 |
| .76  | 53233360   | 53676763 | 54477130                   | 55034538 | 56069097 |
| .77  | 53698690   | 54139720 | 54934963                   | 55488099 | 56513000 |
| .78  | 54159400   | 54597957 | 55387911                   | 55936679 | 56951743 |
| .79  | 54615520   | 55051520 | 55836034                   | 56380332 | 57385407 |
| .80  | 55067100   | 55500460 | 56279393                   | 56819122 | 57814053 |
| .81  | 55514190   | 55944826 | 56718032                   | 57253104 | 58237747 |
| .82  | 55956830   | 56384663 | 57152010                   | 57682342 | 58656558 |
| .83  | 56395070   | 56820024 | 57581379                   | 58106885 | 59070552 |
| .84  | 56828950   | 57250941 | 58006185                   | 58526788 | 59479787 |
| .85  | 57258510   | 57677482 | 58426487                   | 58942112 | 59884324 |
| .86  | 57683790   | 58099674 | 58842334                   | 59352909 | 60284235 |
| .87  | 58104840   | 58517566 | 59253773                   | 59759235 | 60679570 |
| .88  | 58521710   | 58931209 | 59660852                   | 60161137 | 61070395 |
| .89  | 58934420   | 59340645 | 60063627                   | 60558673 | 61456771 |
| .90  | 59343030   | 59745914 | 60462140                   | 60951890 | 61838747 |
| .91  | 59747580   | 60147061 | 60856442                   | 61340844 | 62216390 |
| .92  | 60148100   | 60544132 | 61246575                   | 61725585 | 62589754 |
| .93  | 60544630   | 60937163 | 61632593                   | 62106156 | 62958891 |
| .94  | 60937220   | 61326201 | 62014542                   | 62482611 | 63323859 |
| .95  | 61325900   | 61711281 | 62392459                   | 62854998 | 63684715 |
| .96  | 61710710   | 62092453 | 62766396                   | 63223365 | 64041502 |
| .97  | 62091700   | 62469755 | 63136395                   | 63587757 | 64394281 |
| .98  | 62468890   | 62843225 | 63502498                   | 63948232 | 64743108 |
| .99  | 62842330   | 63212901 | 63864753                   | 64304817 | 65088019 |
| 1.00 | 63212050   | 63578827 | 64223204                   | 64657571 | 65429077 |
| 1.02 | 63940510   | 64299572 | 64928846                   | 65351750 | 66099821 |
| 1.04 | 64654530   | 65005767 | 65619763                   | 66031122 | 66755724 |
| 1.06 | 65354420   | 65697714 | 66296278                   | 66696035 | 67397154 |
| 1.08 | 66040450   | 66375701 | 66958708                   | 67346814 | 68024493 |
| 1.10 | 66712890   | 67040023 | 67607364                   | 67983789 | 68638083 |
| 1.12 | 67372020   | 67690954 | 68242551                   | 68607278 | 69238271 |
| 1.14 | 68018100   | 68328775 | 68864566                   | 69217597 | 69825400 |
| 1.16 | 68651380   | 68953749 | 69473692                   | 69815043 | 70399789 |
| 1.18 | 69272130   | 69566150 | 70070215                   | 70399912 | 70961757 |
| 1.20 | 69880580   | 70166223 | 70654414                   | 70972491 | 71511613 |

Table 1  
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

| $a_2, a_1$ |           |          |          |          |          |
|------------|-----------|----------|----------|----------|----------|
| .8,.2      | .875,.125 | .9,.1    | .95,.05  | .99,.01  | 1,0      |
| 30904502   | 34239293  | 35743308 | 39315313 | 41786435 | 42231980 |
| 31685367   | 35029841  | 36525848 | 40031800 | 42409534 | 42839234 |
| 32455196   | 35806035  | 37292634 | 40731147 | 43019519 | 43434090 |
| 33214184   | 36568234  | 38044135 | 41414136 | 43616996 | 44017077 |
| 33962503   | 37316794  | 38780816 | 42081493 | 44202482 | 44588686 |
|            |           |          |          |          |          |
| 34700338   | 38052044  | 39503091 | 42733888 | 44776487 | 45149375 |
| 35427873   | 38774324  | 40211416 | 43371984 | 45339455 | 45699572 |
| 36145269   | 39483949  | 40906163 | 43996360 | 45891858 | 46239675 |
| 36852712   | 40181223  | 41587734 | 44607603 | 46434061 | 46770058 |
| 37550361   | 40866457  | 42256508 | 45206236 | 46966461 | 47291073 |
|            |           |          |          |          |          |
| 38238377   | 41539937  | 42912860 | 45792751 | 47489418 | 47803048 |
| 38916929   | 42201945  | 43557130 | 46367632 | 48003309 | 48306294 |
| 39586171   | 42852762  | 44189641 | 46931320 | 48508324 | 48801104 |
| 40246264   | 43492654  | 44810751 | 47484234 | 49004850 | 49287753 |
| 40897349   | 44121872  | 45420749 | 48026772 | 49493156 | 49766503 |
|            |           |          |          |          |          |
| 41539584   | 44740684  | 46019964 | 48559293 | 49973544 | 50237601 |
| 42173110   | 45349308  | 46608668 | 49082175 | 50446177 | 50701281 |
| 42798071   | 45948013  | 47187152 | 49595741 | 50911327 | 51157766 |
| 43414615   | 46537002  | 47755681 | 50100311 | 51369210 | 51607267 |
| 44022877   | 47116520  | 48314543 | 50596169 | 51820139 | 52049985 |
|            |           |          |          |          |          |
| 44622989   | 47686773  | 48863968 | 51083638 | 52264164 | 52486112 |
| 45215085   | 48247972  | 49404211 | 51562946 | 52701554 | 52915829 |
| 45799296   | 48800333  | 49935511 | 52034385 | 53132518 | 53339311 |
| 46375751   | 49344041  | 50458097 | 52498171 | 53557193 | 53756724 |
| 46944579   | 49879304  | 50972193 | 52954540 | 53975761 | 54168227 |
|            |           |          |          |          |          |
| 47505897   | 50406297  | 51478013 | 53403730 | 54388367 | 54573972 |
| 48059830   | 50925210  | 51975758 | 53845947 | 54795125 | 54974105 |
| 48606497   | 51436227  | 52465636 | 54281385 | 55196324 | 55368765 |
| 49146014   | 51939511  | 52947840 | 54710236 | 55591942 | 55758085 |
| 49678494   | 52435235  | 53422559 | 55132676 | 55982198 | 56142194 |
|            |           |          |          |          |          |
| 50204051   | 52923565  | 53889967 | 55548897 | 56367209 | 56521216 |
| 50722794   | 53404657  | 54350254 | 55959060 | 56747129 | 56895268 |
| 51234832   | 53878663  | 54803578 | 56363317 | 57122019 | 57264465 |
| 51740273   | 54345740  | 55250106 | 56761835 | 57492014 | 57628917 |
| 52239218   | 54806037  | 55690006 | 57154739 | 57857225 | 57988729 |
|            |           |          |          |          |          |
| 52731775   | 55259681  | 56123415 | 57542196 | 58217808 | 58344003 |
| 53218036   | 55706832  | 56550501 | 57924309 | 58573795 | 58694837 |
| 53698109   | 56147607  | 56971391 | 58301246 | 58925300 | 59041325 |
| 54172086   | 56582151  | 57386241 | 58673088 | 59272426 | 59383559 |
| 54640059   | 57010581  | 57795170 | 59039994 | 59615320 | 59721628 |

Table 1  
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

| $a_2, a_1$ |          |          |                            |          |          |
|------------|----------|----------|----------------------------|----------|----------|
| t          | .5, .5   | .6, .4   | $\frac{2}{3}, \frac{1}{3}$ | .7, .3   | .75, .25 |
| .31        | 26655300 | 27047654 | 27787769                   | 28331071 | 29413895 |
| .32        | 27385100 | 27783585 | 28534544                   | 29085189 | 30180977 |
| .33        | 28107630 | 28511926 | 29273136                   | 29830662 | 30938434 |
| .34        | 28822970 | 29232774 | 30003619                   | 30567580 | 31686416 |
| .35        | 29531190 | 29946202 | 30726115                   | 31296059 | 32425036 |
| .36        | 30232370 | 30652299 | 31440701                   | 32016201 | 33154433 |
| .37        | 30926570 | 31351123 | 32147465                   | 32728117 | 33874743 |
| .38        | 31613860 | 32042763 | 32846513                   | 33431897 | 34586103 |
| .39        | 32294310 | 32727299 | 33537916                   | 34127645 | 35288613 |
| .40        | 32967990 | 33404790 | 34221785                   | 34815461 | 35982426 |
| .41        | 33634970 | 34075330 | 34898183                   | 35495444 | 36667646 |
| .42        | 34295320 | 34738976 | 35567209                   | 36167696 | 37344407 |
| .43        | 34949090 | 35395798 | 36228941                   | 36832298 | 38012822 |
| .44        | 35596360 | 36045884 | 36883469                   | 37489363 | 38673006 |
| .45        | 36237180 | 36689292 | 37530879                   | 38138972 | 39325072 |
| .46        | 36871630 | 37326089 | 38171249                   | 38781216 | 39969147 |
| .47        | 37499770 | 37956353 | 38804655                   | 39416193 | 40605328 |
| .48        | 38121660 | 38580154 | 39431182                   | 40043989 | 41233735 |
| .49        | 38737360 | 39197546 | 40050906                   | 40664686 | 41854467 |
| .50        | 39346930 | 39808616 | 40663912                   | 41278378 | 42467636 |
| .51        | 39950440 | 40413412 | 41270274                   | 41885149 | 43073353 |
| .52        | 40547940 | 41012007 | 41870059                   | 42485085 | 43671717 |
| .53        | 41139500 | 41604461 | 42463357                   | 43078270 | 44262816 |
| .54        | 41725170 | 42190852 | 43050230                   | 43664773 | 44846772 |
| .55        | 42305020 | 42771238 | 43630756                   | 44244699 | 45423671 |
| .56        | 42879090 | 43345664 | 44205016                   | 44818116 | 45993616 |
| .57        | 43447460 | 43914219 | 44773070                   | 45385105 | 46556704 |
| .58        | 44010160 | 44476942 | 45334998                   | 45945736 | 47113021 |
| .59        | 44567270 | 45033909 | 45890858                   | 46500095 | 47662665 |
| .60        | 45118840 | 45585173 | 46440731                   | 47048263 | 48205725 |
| .61        | 45664910 | 46130799 | 46984675                   | 47590303 | 48742296 |
| .62        | 46205560 | 46670841 | 47522767                   | 48126296 | 49272466 |
| .63        | 46740820 | 47205357 | 48055071                   | 48656310 | 49796320 |
| .64        | 47270760 | 47734407 | 48581649                   | 49180432 | 50313947 |
| .65        | 47795420 | 48258049 | 49102571                   | 49698716 | 50825420 |
| .66        | 48314870 | 48776333 | 49617898                   | 50211238 | 51330837 |
| .67        | 48829140 | 49289322 | 50127695                   | 50718073 | 51830271 |
| .68        | 49338300 | 49797072 | 50632022                   | 51219288 | 52323811 |
| .69        | 49842390 | 50299627 | 51130951                   | 51714947 | 52811532 |
| .70        | 50341470 | 50797057 | 51624531                   | 52205117 | 53293512 |



Table 1  
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

| $a_2, a_1$ |            |          |          |          |          |
|------------|------------|----------|----------|----------|----------|
| .8, .2     | .875, .125 | .9, .1   | .95, .05 | .99, .01 | 1, 0     |
| 00622552   | 00751637   | 00827521 | 01132120 | 02363402 | 05637184 |
| 01240281   | 01494752   | 01643737 | 02235194 | 04462235 | 07965566 |
| 01853219   | 02229476   | 02448798 | 03310213 | 06340272 | 09747674 |
| 02461403   | 02955933   | 03242893 | 04358204 | 08033210 | 11246290 |
| 03064878   | 03674233   | 04026239 | 05380110 | 09570210 | 12563292 |
|            |            |          |          |          |          |
| 03663696   | 04384517   | 04799013 | 06376875 | 10975230 | 13750975 |
| 04257889   | 05086866   | 05561415 | 07349374 | 12267953 | 14840432 |
| 04847507   | 05781421   | 06313627 | 08298478 | 13464614 | 15851940 |
| 05432587   | 06468292   | 07055811 | 09224988 | 14578637 | 16799595 |
| 06013164   | 07147575   | 07788187 | 10129720 | 15621237 | 17693670 |
|            |            |          |          |          |          |
| 06589297   | 07819382   | 08510892 | 11013393 | 16601672 | 18541928 |
| 07161002   | 08483835   | 09224123 | 11876763 | 17527757 | 19350404 |
| 07728344   | 09141011   | 09928039 | 12720504 | 18406023 | 20123902 |
| 08291341   | 09791040   | 10622785 | 13545298 | 19241995 | 20866320 |
| 08850052   | 10434010   | 11308555 | 14351792 | 20040332 | 21580875 |
|            |            |          |          |          |          |
| 09404497   | 11070023   | 11985484 | 15140605 | 20805043 | 22270257 |
| 09954734   | 11699175   | 12653732 | 15912315 | 21539465 | 22936743 |
| 10500784   | 12321571   | 13313452 | 16667510 | 22246567 | 23582282 |
| 11042705   | 12937291   | 13964796 | 17406712 | 22928846 | 24208557 |
| 11580504   | 13546441   | 14607895 | 18130495 | 23588494 | 24817034 |
|            |            |          |          |          |          |
| 12643947   | 14745392   | 15869979 | 19533695 | 24847151 | 25985584 |
| 13691407   | 15919140   | 17100796 | 20880955 | 26035077 | 27096549 |
| 14723178   | 17068372   | 18301415 | 22175776 | 27161993 | 28156794 |
| 15739530   | 18193766   | 19472857 | 23421443 | 28235589 | 29171897 |
| 16740739   | 19295988   | 20616102 | 24620977 | 29262051 | 30146462 |
|            |            |          |          |          |          |
| 17727068   | 20375651   | 21732100 | 25777196 | 30246433 | 31084346 |
| 18698782   | 21433369   | 22821750 | 26892675 | 31192938 | 31988817 |
| 19656163   | 22469758   | 23885932 | 27969832 | 32105057 | 32862674 |
| 20599425   | 23485371   | 24925474 | 29010896 | 32985779 | 33708336 |
| 21528848   | 24480778   | 25941196 | 30017954 | 33837696 | 34527913 |
|            |            |          |          |          |          |
| 22444653   | 25456500   | 26933858 | 30992912 | 34662951 | 35323257 |
| 23347094   | 26413083   | 27904216 | 31937555 | 35463553 | 36096006 |
| 24236404   | 27351019   | 28852980 | 32853564 | 36241151 | 36847615 |
| 25112804   | 28270809   | 29780841 | 33742471 | 36997296 | 37579386 |
| 25976511   | 29172920   | 30688466 | 34605715 | 37733286 | 38292490 |
|            |            |          |          |          |          |
| 26827767   | 30057815   | 31576492 | 35444631 | 38450367 | 38987982 |
| 27666772   | 30925949   | 32445527 | 36260474 | 39149589 | 39666820 |
| 28493741   | 31777749   | 33296170 | 37054418 | 39831953 | 40329876 |
| 29308886   | 32613636   | 34128986 | 37827535 | 40498355 | 40977945 |
| 30112404   | 33434019   | 34944522 | 38580854 | 41149611 | 41611756 |

Table 1  
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

| t    | $a_2, a_1$ |          |                            |          |          |
|------|------------|----------|----------------------------|----------|----------|
|      | .5, .5     | .6, .4   | $\frac{2}{3}, \frac{1}{3}$ | .7, .3   | .75, .25 |
| .005 | 00498750   | 00509010 | 00528896                   | 00543955 | 00575466 |
| .01  | 00995020   | 01015346 | 01054773                   | 01084654 | 01147062 |
| .015 | 01488810   | 01519052 | 01577693                   | 01622152 | 01714889 |
| .02  | 01980130   | 02020149 | 02097692                   | 02156449 | 02278938 |
| .025 | 02469010   | 02518631 | 02614774                   | 02687584 | 02839252 |
| .03  | 02955450   | 03014547 | 03128965                   | 03215582 | 03395869 |
| .035 | 03439460   | 03507871 | 03640266                   | 03740464 | 03948795 |
| .04  | 03921060   | 03998643 | 04148721                   | 04262225 | 04498082 |
| .045 | 04400250   | 04486862 | 04654316                   | 04780900 | 05043733 |
| .05  | 04877060   | 04972544 | 05157095                   | 05296521 | 05585790 |
| .055 | 05351490   | 05455719 | 05657052                   | 05809095 | 06124268 |
| .06  | 05823550   | 05936372 | 06154223                   | 06318651 | 06659209 |
| .065 | 06293250   | 06414542 | 06648616                   | 06825189 | 07190632 |
| .07  | 06760620   | 06890232 | 07140245                   | 07328737 | 07718554 |
| .075 | 07225650   | 07363438 | 07629123                   | 07829324 | 08243012 |
| .08  | 07688370   | 07834209 | 08115273                   | 08326950 | 08764033 |
| .085 | 08148770   | 08302525 | 08598722                   | 08821648 | 09281627 |
| .09  | 08606880   | 08768418 | 09079462                   | 09313447 | 09795844 |
| .095 | 09062710   | 09231892 | 09557529                   | 09802345 | 10306685 |
| .10  | 09516260   | 09692971 | 10032922                   | 10288367 | 10814191 |
| .11  | 10416590   | 10607969 | 10975769                   | 11251852 | 11819275 |
| .12  | 11307960   | 11513484 | 11908151                   | 12204059 | 12811291 |
| .13  | 12190460   | 12409670 | 12830180                   | 13145125 | 13790429 |
| .14  | 13064180   | 13296574 | 13741973                   | 14075183 | 14756880 |
| .15  | 13929200   | 14174318 | 14643665                   | 14994399 | 15710816 |
| .16  | 14785620   | 15042992 | 15535349                   | 15902883 | 16652434 |
| .17  | 15633520   | 15902704 | 16417172                   | 16800774 | 17581926 |
| .18  | 16472980   | 16753526 | 17289226                   | 17688203 | 18499442 |
| .19  | 17304090   | 17595572 | 18151625                   | 18565323 | 19405159 |
| .20  | 18126920   | 18428925 | 19004502                   | 19432245 | 20299268 |
| .21  | 18941570   | 19253688 | 19847958                   | 20289102 | 21181921 |
| .22  | 19748120   | 20069921 | 20682098                   | 21136022 | 22053292 |
| .23  | 20546640   | 20877749 | 21507029                   | 21973117 | 22913540 |
| .24  | 21337210   | 21677242 | 22322870                   | 22800534 | 23762830 |
| .25  | 22119920   | 22468494 | 23129720                   | 23618374 | 24601312 |
| .26  | 22894840   | 23251585 | 23927687                   | 24426771 | 25429148 |
| .27  | 23662050   | 24026612 | 24716859                   | 25225832 | 26246490 |
| .28  | 24421620   | 24793631 | 25497364                   | 26015685 | 27053487 |
| .29  | 25173640   | 25552769 | 26269271                   | 26796426 | 27850289 |
| .30  | 25918180   | 26304081 | 27032711                   | 27568185 | 28637046 |

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Table 1  
 $P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 \leq t]$

| $a_2, a_1$ |          |          |                            |          |          |
|------------|----------|----------|----------------------------|----------|----------|
| t          | .5,.5    | .6,.4    | $\frac{2}{3}, \frac{1}{3}$ | .7,.3    | .75,.25  |
| 4.30       | 98643140 | 98510648 | 98283675                   | 98134065 | 97872195 |
| 4.35       | 98709320 | 98579092 | 98356188                   | 98209364 | 97952495 |
| 4.40       | 98772270 | 98644341 | 98425545                   | 98281518 | 98029651 |
| 4.45       | 98832140 | 98706548 | 98491890                   | 98350667 | 98103796 |
| 4.50       | 98889100 | 98765857 | 98555356                   | 98416942 | 98175050 |
| 4.55       | 98943280 | 98822405 | 98616075                   | 98480465 | 98243533 |
| 4.60       | 98994820 | 98876323 | 98674168                   | 98541358 | 98309359 |
| 4.65       | 99043840 | 98927735 | 98729755                   | 98599733 | 98372634 |
| 4.70       | 99090470 | 98976760 | 98782946                   | 98655699 | 98433463 |
| 4.75       | 99134830 | 99023510 | 98833849                   | 98709360 | 98491945 |
| 4.80       | 99177030 | 99068092 | 98882565                   | 98760814 | 98548174 |
| 4.85       | 99217160 | 99110609 | 98929192                   | 98810156 | 98602240 |
| 4.90       | 99255340 | 99151158 | 98973821                   | 98857476 | 98654232 |
| 4.95       | 99291660 | 99189832 | 99016543                   | 98902860 | 98704232 |
| 5.00       | 99326210 | 99226718 | 99057440                   | 98946390 | 98752318 |
| 5.25       | 99475250 | 99387139 | 99237254                   | 99138867 | 98966622 |
| 5.50       | 99591320 | 99513898 | 99382084                   | 99295420 | 99143279 |
| 5.75       | 99681720 | 99614149 | 99498904                   | 99422940 | 99289104 |
| 6.00       | 99752120 | 99693502 | 99593255                   | 99526952 | 99409627 |
| 6.50       | 99849660 | 99806207 | 99731320                   | 99681339 | 99591948 |
| 7.00       | 99908810 | 99877154 | 99821985                   | 99784728 | 99717245 |
| 8.00       | 99966450 | 99950295 | 99921266                   | 99901068 | 99863371 |
| 9.00       | 99987660 | 99979729 | 99964897                   | 99954195 | 99933531 |
| 10.00      | 99995460 | 99991677 | 99984251                   | 99978669 | 99967489 |

Table 1  
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

| $a_2, a_1$ |           |          |          |          |          |
|------------|-----------|----------|----------|----------|----------|
| .8,.2      | .875,.125 | .9,.1    | .95,.05  | .99,.01  | 1,0      |
| 97574718   | 97082531  | 96909701 | 96554339 | 96262571 | 96188759 |
| 97660761   | 97177801  | 97008064 | 96658789 |          | 96299102 |
| 97743636   | 97269838  | 97103169 | 96759938 |          | 96406104 |
| 97823466   | 97358755  | 97195128 | 96857895 |          | 96509872 |
| 97900369   | 97444663  | 97284054 | 96952765 |          | 96610511 |
| 97974459   | 97527674  | 97370051 | 97044648 |          | 96708122 |
| 98045842   | 97607887  | 97453221 | 97133653 |          | 96802801 |
| 98114624   | 97685402  | 97533662 | 97219872 |          | 96894642 |
| 98180904   | 97760316  | 97611469 | 97303396 |          | 96983735 |
| 98244776   | 97832720  | 97686729 | 97384313 |          | 97070167 |
| 98306333   | 97902702  | 97759536 | 97462710 |          | 97154023 |
| 98365662   | 97970346  | 97829970 | 97538670 |          | 97235383 |
| 98422848   | 98035736  | 97898114 | 97612272 |          | 97314327 |
| 98477971   | 98098950  | 97964045 | 97683593 |          | 97390930 |
| 98531109   | 98160062  | 98027839 | 97752701 |          | 97465265 |
| 98769535   | 98436583  | 98317194 | 98067610 |          | 97805320 |
| 98968346   | 98670471  | 98562960 | 98337119 |          | 98098350 |
| 99134333   | 98868527  | 98771937 | 98568037 |          | 98351132 |
| 99273073   | 99036418  | 98949818 | 98766083 |          | 98569410 |
| 99486379   | 99299858  | 99230612 | 99082173 |          | 98921253 |
| 99636234   | 99490152  | 99435105 | 99315858 |          | 99184901 |
| 99816454   | 99728130  | 99693816 | 99617861 |          | 99532225 |
| 99906792   | 99854147  | 99833048 | 99785304 |          | 99730018 |
| 99952424   | 99921369  | 99908526 | 99878809 |          | 99843457 |

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

| $a_3, a_2, a_1$ |   |            |            |            |            |
|-----------------|---|------------|------------|------------|------------|
| t               | $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ | .4, .3, .3 | .4, .4, .2 | .5, .3, .2 | .6, .2, .2 |
| .01             | 00136861                                | 00138896   | 00147199   | 00151975   | 00169689   |
| .02             | 00383822                                | 00389282   | 00412216   | 00425451   | 00474413   |
| .03             | 00698916                                | 00708662   | 00749801   | 00773620   | 00861521   |
| .04             | 01066521                                | 01081164   | 01143008   | 01178934   | 01311181   |
| .05             | 01477295                                | 01497298   | 01581685   | 01630868   | 01811473   |
| .06             | 01924750                                | 01950469   | 02058774   | 02122104   | 02354095   |
| .07             | 02403997                                | 02435709   | 02568953   | 02647123   | 02932783   |
| .08             | 02911165                                | 02949081   | 03108009   | 03201562   | 03542578   |
| .09             | 03443076                                | 03487356   | 03672485   | 03781823   | 04179415   |
| .10             | 03997053                                | 04047815   | 04259472   | 04384900   | 04839882   |
| .11             | 04570803                                | 04628124   | 04866481   | 05008211   | 05521059   |
| .12             | 05162325                                | 05226254   | 05491345   | 05649509   | 06220418   |
| .13             | 05769859                                | 05840416   | 06132161   | 06306819   | 06935737   |
| .14             | 06391834                                | 06469014   | 06787233   | 06978394   | 07665051   |
| .15             | 07026844                                | 07110622   | 07455048   | 07662663   | 08406610   |
| .16             | 07673618                                | 07763948   | 08134239   | 08358224   | 09158843   |
| .17             | 08330998                                | 08427822   | 08823571   | 09063786   | 09920331   |
| .18             | 08997932                                | 09101174   | 09521917   | 09778195   | 10689789   |
| .19             | 09673450                                | 09783022   | 10228248   | 10500393   | 11466051   |
| .20             | 10356664                                | 10472469   | 10941618   | 11229403   | 12248051   |
| .21             | 11046750                                | 11168677   | 11661158   | 11964331   | 13034814   |
| .22             | 11742948                                | 11870881   | 12386069   | 12704354   | 13825448   |
| .23             | 12444552                                | 12578362   | 13115609   | 13448712   | 14619134   |
| .24             | 13150904                                | 13290461   | 13849091   | 14196705   | 15415117   |
| .25             | 13861393                                | 14006559   | 14585878   | 14947680   | 16212698   |
| .26             | 14575447                                | 14726077   | 15325378   | 15701021   | 17011238   |
| .27             | 15292531                                | 15448476   | 16067037   | 16456175   | 17810142   |
| .28             | 16012146                                | 16173257   | 16810345   | 17212621   | 18608865   |
| .29             | 16733822                                | 16899941   | 17554818   | 17969861   | 19406896   |
| .30             | 17457120                                | 17628091   | 18300012   | 18727450   | 20203771   |
| .31             | 18181627                                | 18357289   | 19045508   | 19484964   | 20999051   |
| .32             | 18906954                                | 19087142   | 19790913   | 20241999   | 21792333   |
| .33             | 19632737                                | 19817290   | 20535861   | 20998188   | 22583251   |
| .34             | 20358631                                | 20547386   | 21280010   | 21753182   | 23371451   |
| .35             | 21084313                                | 21277103   | 22023037   | 22506667   | 24156619   |
| .36             | 21809479                                | 22006139   | 22764643   | 23258337   | 24938459   |
| .37             | 22533841                                | 22734205   | 23504547   | 24007906   | 25716692   |
| .38             | 23257128                                | 23461029   | 24242487   | 24755107   | 26491072   |
| .39             | 23979085                                | 24186362   | 24978208   | 25499707   | 27261362   |
| .40             | 24699471                                | 24909958   | 25711486   | 26241466   | 28027344   |

Table 2

$$P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 + a_3^{(3)}\chi_1^2 \leq t]$$

| $a_3, a_2, a_1$ |            |            |            |            |              |
|-----------------|------------|------------|------------|------------|--------------|
| t               | .5, .4, .1 | .6, .3, .1 | .7, .2, .1 | .8, .1, .1 | .9, .05, .05 |
| .01             | 00185354   | 00195289   | 00221126   | 00291125   | 00538290     |
| .02             | 00516804   | 00544222   | 00615346   | 00806304   | 01462629     |
| .03             | 00935993   | 00985162   | 01112332   | 01450702   | 02583012     |
| .04             | 01420799   | 01494686   | 01685248   | 02187747   | 03825329     |
| .05             | 01957883   | 02058680   | 02317899   | 02995303   | 05145697     |
| .06             | 02537983   | 02667320   | 02998998   | 03857970   | 06514772     |
| .07             | 03154112   | 03313223   | 03720053   | 04764226   | 07911739     |
| .08             | 03800801   | 03990564   | 04474383   | 05705063   | 09321344     |
| .09             | 04473534   | 04694589   | 05256547   | 06673226   | 10732192     |
| .10             | 05168606   | 05421351   | 06062030   | 07662755   | 12135693     |
| .11             | 05882839   | 06167509   | 06887004   | 08668671   | 13525348     |
| .12             | 06613505   | 06930171   | 07728204   | 09686777   | 14896260     |
| .13             | 07358234   | 07706846   | 08582787   | 10713503   | 16244770     |
| .14             | 08114973   | 08495343   | 09448288   | 11745788   | 17568187     |
| .15             | 08881871   | 09293739   | 10322548   | 12781003   | 18864589     |
| .16             | 09657328   | 10100322   | 11203638   | 13816871   | 20132649     |
| .17             | 10439866   | 10913571   | 12089877   | 14851430   | 21371540     |
| .18             | 11228206   | 11732121   | 12979758   | 15882976   | 22580803     |
| .19             | 12021196   | 12554770   | 13871944   | 16910028   | 23760295     |
| .20             | 12817774   | 13380420   | 14765233   | 17931307   | 24910102     |
| .21             | 13616984   | 14208066   | 15658543   | 18945700   | 26030510     |
| .22             | 14417978   | 15036826   | 16550928   | 19952248   | 27121957     |
| .23             | 15219976   | 15865892   | 17441520   | 20950122   | 28184978     |
| .24             | 16022284   | 16694549   | 18329545   | 21938606   | 29220211     |
| .25             | 16824259   | 17522131   | 19214317   | 22917098   | 30228352     |
| .26             | 17625297   | 18348014   | 20095202   | 23885075   | 31210143     |
| .27             | 18424890   | 19171684   | 20971652   | 24842099   | 32166360     |
| .28             | 19222553   | 19992645   | 21843161   | 25787804   | 33097788     |
| .29             | 20017837   | 20810443   | 22709277   | 26721891   | 34005228     |
| .30             | 20810355   | 21624664   | 23569605   | 27644112   | 34889491     |
| .31             | 21599746   | 22434948   | 24423791   | 28554270   | 35751354     |
| .32             | 22385658   | 23240964   | 25271508   | 29452221   | 36591620     |
| .33             | 23167799   | 24042421   | 26112477   | 30337853   | 37411041     |
| .34             | 23945886   | 24839025   | 26946454   | 31211092   | 38210384     |
| .35             | 24719691   | 25630542   | 27773210   | 32071896   | 38990368     |
| .36             | 25488984   | 26416751   | 28592548   | 32920256   | 39751714     |
| .37             | 26253542   | 27197443   | 29404318   | 33756180   | 40495102     |
| .38             | 27013176   | 27972458   | 30208360   | 34579702   | 41221199     |
| .39             | 27767752   | 28741615   | 31004545   | 35390878   | 41930643     |
| .40             | 28517088   | 29504792   | 31792795   | 36189780   | 42624046     |



Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

| t   | $a_3, a_2, a_1$                         |            |            |            |            |
|-----|---|------------|------------|------------|------------|
|     | $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ | .4, .3, .3 | .4, .4, .2 | .5, .3, .2 | .6, .2, .2 |
| .41 | 25418059                                | 25631592   | 26442100   | 26980170   | 28788818   |
| .42 | 26134635                                | 26351053   | 27169847   | 27715618   | 29545604   |
| .43 | 26848998                                | 27068139   | 27894538   | 28447628   | 30297532   |
| .44 | 27560957                                | 27782664   | 28615995   | 29176018   | 31044447   |
| .45 | 28270332                                | 28494447   | 29334052   | 29900625   | 31786204   |
| .46 | 28976955                                | 29203319   | 30048550   | 30621297   | 32522675   |
| .47 | 29680666                                | 29909123   | 30759347   | 31337902   | 33253742   |
| .48 | 30381315                                | 30611717   | 31466309   | 32050290   | 33979296   |
| .49 | 31078760                                | 31310952   | 32169301   | 32758360   | 34699235   |
| .50 | 31772869                                | 32006709   | 32868215   | 33461981   | 35413480   |
| .51 | 32463517                                | 32698856   | 33562934   | 34161059   | 36121944   |
| .52 | 33150586                                | 33387275   | 34253362   | 34855500   | 36824553   |
| .53 | 33833966                                | 34071867   | 34939399   | 35545209   | 37521245   |
| .54 | 34513553                                | 34752528   | 35620959   | 36230089   | 38211970   |
| .55 | 35189250                                | 35429163   | 36297965   | 36910093   | 38896669   |
| .56 | 35860966                                | 36101682   | 36970337   | 37585143   | 39575308   |
| .57 | 36528617                                | 36770008   | 37638009   | 38255163   | 40247847   |
| .58 | 37192123                                | 37434055   | 38300917   | 38920105   | 40914255   |
| .59 | 37851410                                | 38093758   | 38959001   | 39579916   | 41574504   |
| .60 | 38506409                                | 38749054   | 39612215   | 40234547   | 42228573   |
| .61 | 39157056                                | 39399868   | 40260504   | 40883970   | 42876453   |
| .62 | 39803292                                | 40046160   | 40903827   | 41528126   | 43518126   |
| .63 | 40445062                                | 40687862   | 41542146   | 42167000   | 44153597   |
| .64 | 41082315                                | 41324942   | 42175426   | 42800557   | 44782850   |
| .65 | 41715005                                | 41957346   | 42803637   | 43428762   | 45405886   |
| .66 | 42343088                                | 42585032   | 43426748   | 44051610   | 46022716   |
| .67 | 42966526                                | 43207969   | 44044739   | 44669081   | 46633349   |
| .68 | 43585282                                | 43826122   | 44657591   | 45281153   | 47237788   |
| .69 | 44199325                                | 44439466   | 45265283   | 45887817   | 47836057   |
| .70 | 44808626                                | 45047967   | 45867801   | 46489066   | 48428163   |
| .71 | 45413158                                | 45651604   | 46465138   | 47084897   | 49014135   |
| .72 | 46012899                                | 46250362   | 47057286   | 47675313   | 49593991   |
| .73 | 46607828                                | 46844212   | 47644230   | 48260303   | 50167749   |
| .74 | 47197928                                | 47433154   | 48225979   | 48839880   | 50735444   |
| .75 | 47783184                                | 48017162   | 48802526   | 49414040   | 51297101   |
| .76 | 48363583                                | 48596230   | 49373875   | 49982798   | 51852746   |
| .77 | 48939115                                | 49170354   | 49940025   | 50546154   | 52402410   |
| .78 | 49509772                                | 49739529   | 50500985   | 51104139   | 52946141   |
| .79 | 50075549                                | 50303749   | 51056763   | 51656746   | 53483951   |
| .80 | 50636441                                | 50863009   | 51607364   | 52203992   | 54015889   |

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

| $a_3, a_2, a_1$ |            |            |            |            |              |
|-----------------|------------|------------|------------|------------|--------------|
| t               | .5, .4, .1 | .6, .3, .1 | .7, .2, .1 | .8, .1, .1 | .9, .05, .05 |
| .41             | 29261073   | 30261851   | 32572985   | 36976487   | 43302001     |
| .42             | 29999565   | 31012688   | 33345058   | 37751116   | 43965086     |
| .43             | 30732463   | 31757188   | 34108965   | 38513765   | 44613827     |
| .44             | 31459680   | 32495286   | 34864654   | 39264561   | 45248755     |
| .45             | 32181120   | 33226905   | 35612090   | 40003638   | 45870363     |
| .46             | 32896701   | 33951978   | 36351254   | 40731134   | 46479134     |
| .47             | 33606395   | 34670450   | 37082142   | 41447193   | 47075518     |
| .48             | 34310096   | 35382276   | 37804743   | 42151970   | 47659955     |
| .49             | 35007792   | 36087437   | 38519063   | 42845618   | 48232858     |
| .50             | 35699433   | 36785892   | 39225136   | 43528299   | 48794624     |
| .51             | 36384986   | 37477615   | 39922965   | 44200178   | 49345636     |
| .52             | 37064424   | 38162604   | 40612586   | 44861419   | 49886251     |
| .53             | 37737729   | 38840853   | 41294036   | 45512196   | 50416820     |
| .54             | 38404870   | 39512356   | 41967352   | 46152668   | 50937673     |
| .55             | 39065871   | 40177118   | 42632581   | 46783007   | 51449122     |
| .56             | 39720705   | 40835144   | 43289764   | 47403388   | 51951467     |
| .57             | 40369371   | 41486450   | 43938970   | 48013982   | 52444999     |
| .58             | 41011878   | 42131064   | 44580247   | 48614954   | 52929994     |
| .59             | 41648229   | 42768989   | 45213660   | 49206482   | 53406715     |
| .60             | 42278443   | 43400255   | 45839277   | 49788719   | 53875408     |
| .61             | 42902535   | 44024894   | 46457141   | 50361848   | 54336320     |
| .62             | 43520511   | 44642936   | 47067346   | 50926033   | 54789670     |
| .63             | 44132413   | 45254416   | 47669960   | 51481427   | 55235687     |
| .64             | 44738246   | 45859357   | 48265042   | 52028203   | 55674577     |
| .65             | 45338024   | 46457816   | 48852678   | 52566517   | 56106542     |
| .66             | 45931807   | 47049823   | 49432942   | 53096535   | 56531774     |
| .67             | 46519608   | 47635422   | 50005904   | 53618403   | 56950459     |
| .68             | 47101464   | 48214645   | 50571648   | 54132287   | 57362769     |
| .69             | 47677392   | 48787556   | 51130249   | 54638328   | 57768883     |
| .70             | 48247433   | 49354193   | 51681795   | 55136683   | 58158950     |
| .71             | 48811634   | 49914608   | 52226353   | 55627503   | 58563131     |
| .72             | 49370033   | 50468847   | 52764018   | 56110922   | 58951592     |
| .73             | 49922656   | 51016949   | 53294859   | 56587093   | 59334454     |
| .74             | 50469548   | 51558987   | 53818961   | 57056144   | 59711860     |
| .75             | 51010757   | 52095003   | 54336409   | 57518226   | 60083946     |
| .76             | 51546305   | 52625046   | 54847242   | 57973458   | 60450845     |
| .77             | 52076258   | 53149166   | 55351577   | 58421990   | 60812676     |
| .78             | 52600650   | 53667421   | 55849568   | 58863939   | 61169552     |
| .79             | 53119520   | 54179872   | 56341189   | 59299440   | 61521598     |
| .80             | 53632902   | 54686576   | 56826519   | 59728614   | 61868910     |

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

| t    | $a_3, a_2, a_1$                         |            |            |            |            |
|------|---|------------|------------|------------|------------|
|      | $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ | .4, .3, .3 | .4, .4, .2 | .5, .3, .2 | .6, .2, .2 |
| .81  | 51192447                                | 51417315   | 52152804   | 52745897   | 54541999   |
| .82  | 51743567                                | 51966667   | 52693093   | 53282484   | 55062306   |
| .83  | 52289802                                | 52511074   | 53228249   | 53813769   | 55576864   |
| .84  | 52831156                                | 53050534   | 53758278   | 54339775   | 56085699   |
| .85  | 53367634                                | 53585057   | 54283210   | 54860515   | 56588862   |
| .86  | 53899243                                | 54114654   | 54803052   | 55376028   | 57086394   |
| .87  | 54425991                                | 54639338   | 55317829   | 55886328   | 57578335   |
| .88  | 54947888                                | 55159123   | 55827560   | 56391446   | 58064730   |
| .89  | 55464945                                | 55674003   | 56332271   | 56891406   | 58545624   |
| .90  | 55977173                                | 56184008   | 56831984   | 57386236   | 59021066   |
| .91  | 56484586                                | 56689157   | 57326718   | 57875956   | 59491102   |
| .92  | 56987199                                | 57189460   | 57816495   | 58360619   | 59955772   |
| .93  | 57485028                                | 57684929   | 58301360   | 58840227   | 60415134   |
| .94  | 57978089                                | 58175596   | 58781313   | 59314832   | 60869226   |
| .95  | 58466400                                | 58661464   | 59256393   | 59784460   | 61318094   |
| .96  | 58949979                                | 59142579   | 59726633   | 60249150   | 61761789   |
| .97  | 59428846                                | 59618932   | 60192066   | 60708915   | 62200358   |
| .98  | 59903022                                | 60090569   | 60652695   | 61163810   | 62633758   |
| .99  | 60372528                                | 60557499   | 61108584   | 61613850   | 63062329   |
| 1.00 | 60837386                                | 61019754   | 61559739   | 62059099   | 63485810   |
| 1.01 | 61297618                                | 61477356   | 62006200   | 62499557   | 63904375   |
| 1.02 | 61753248                                | 61930330   | 62448000   | 62935281   | 64318042   |
| 1.03 | 62204301                                | 62378704   | 62885168   | 63366299   | 64726884   |
| 1.04 | 62650801                                | 62822497   | 63317736   | 63792658   | 65130942   |
| 1.05 | 63092773                                | 63261745   | 63745738   | 64214379   | 65530270   |
| 1.06 | 63530243                                | 63696459   | 64169207   | 64631514   | 65924902   |
| 1.07 | 63963238                                | 64126692   | 64588172   | 65044090   | 66314890   |
| 1.08 | 64391784                                | 64552457   | 65002680   | 65452139   | 66700298   |
| 1.09 | 64815909                                | 64973793   | 65412757   | 65855714   | 67081163   |
| 1.10 | 65235640                                | 65390702   | 65818428   | 66254849   | 67457533   |
| 1.11 | 65651006                                | 65803249   | 66219730   | 66649572   | 67829455   |
| 1.12 | 66062036                                | 66211437   | 66616712   | 67039930   | 68196983   |
| 1.13 | 66468758                                | 66615317   | 67009396   | 67425951   | 68560155   |
| 1.14 | 66871201                                | 67014915   | 67397821   | 67807678   | 68919025   |
| 1.15 | 67269396                                | 67410241   | 67782027   | 68185154   | 69273637   |
| 1.16 | 67663371                                | 67801353   | 68162030   | 68558412   | 69624044   |
| 1.17 | 68053157                                | 68188262   | 68537887   | 68927487   | 69970291   |
| 1.18 | 68438785                                | 68571019   | 68909630   | 69292427   | 70312425   |
| 1.19 | 68820285                                | 68949635   | 69277279   | 69653272   | 70650479   |
| 1.20 | 69197687                                | 69324162   | 69640892   | 70010045   | 70984513   |

Table 2

$$P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 + a_3^{(3)}\chi_1^2 \leq t]$$

| $a_3, a_2, a_1$ |            |            |            |            |              |
|-----------------|------------|------------|------------|------------|--------------|
| t               | .5, .4, .1 | .6, .3, .1 | .7, .2, .1 | .8, .1, .1 | .9, .05, .05 |
| .81             | 54140868   | 55187554   | 57305772   | 60151583   | 62211604     |
| .82             | 54643444   | 55682901   | 57778892   | 60568465   | 62549777     |
| .83             | 55140679   | 56172653   | 58245896   | 60979382   | 62883533     |
| .84             | 55632634   | 56656870   | 58707074   | 61384440   | 63212965     |
| .85             | 56119326   | 57135607   | 59162413   | 61783765   | 63538158     |
| .86             | 56600826   | 57608924   | 59611912   | 62177450   | 63859216     |
| .87             | 57077162   | 58076866   | 60055737   | 62565623   | 64176207     |
| .88             | 57548397   | 58539502   | 60493964   | 62948377   | 64489220     |
| .89             | 58014572   | 58996870   | 60926613   | 63325800   | 64798344     |
| .90             | 58475729   | 59449039   | 61353748   | 63698022   | 65103643     |
| .91             | 58931909   | 59896051   | 61775540   | 64065132   | 65405211     |
| .92             | 59383186   | 60337977   | 62191990   | 64427214   | 65703102     |
| .93             | 59829572   | 60774863   | 62603195   | 64784372   | 65997395     |
| .94             | 60271142   | 61206764   | 63009206   | 65136697   | 66288154     |
| .95             | 60707920   | 61633735   | 63410077   | 65484279   | 66575457     |
| .96             | 61139969   | 62055828   | 63805981   | 65827209   | 66859353     |
| .97             | 61567324   | 62473101   | 64196876   | 66165563   | 67139916     |
| .98             | 61990049   | 62885595   | 64582820   | 66499439   | 67417198     |
| .99             | 62408166   | 63293377   | 64964018   | 66828909   | 67691264     |
| 1.00            | 62821748   | 63696493   | 65340437   | 67154057   | 67962173     |
| 1.01            | 63230807   | 64094997   | 65712017   | 67474961   | 68229982     |
| 1.02            | 63635410   | 64488869   | 66079097   | 67791702   | 68494737     |
| 1.03            | 64035595   | 64878287   | 66441622   | 68104347   | 68756496     |
| 1.04            | 64431419   | 65263296   | 66799599   | 68412974   | 69015306     |
| 1.05            | 64822923   | 65643885   | 67153107   | 68717655   | 69271226     |
| 1.06            | 65210129   | 66020017   | 67502269   | 69018455   | 69524288     |
| 1.07            | 65593103   | 66391921   | 67847022   | 69315451   | 69774558     |
| 1.08            | 65971891   | 66759610   | 68187652   | 69608708   | 70022082     |
| 1.09            | 66346531   | 67123063   | 68524080   | 69898292   | 70266891     |
| 1.10            | 66717059   | 67482267   | 68856148   | 70184249   | 70509030     |
| 1.11            | 67083524   | 67837362   | 69184379   | 70466675   | 70748562     |
| 1.12            | 67445970   | 68188485   | 69508614   | 70745599   | 70985500     |
| 1.13            | 67804431   | 68535599   | 69828629   | 71021105   | 71219911     |
| 1.14            | 68158967   | 68878695   | 70144915   | 71293239   | 71451816     |
| 1.15            | 68509593   | 69217818   | 70457365   | 71562055   | 71681263     |
| 1.16            | 68856384   | 69553117   | 70765817   | 71827622   | 71908298     |
| 1.17            | 69199339   | 69884618   | 71070698   | 72089978   | 72132935     |
| 1.18            | 69538530   | 70212270   | 71371788   | 72349200   | 72355232     |
| 1.19            | 69873995   | 70536169   | 71669267   | 72605313   | 72575202     |
| 1.20            | 70205769   | 70856355   | 71963213   | 72858382   | 72792905     |

Table 2

$$P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 + a_3^{(3)}\chi_1^2 \leq t]$$

| t    | $a_3, a_2, a_1$                         |            |            |            |            |
|------|---|------------|------------|------------|------------|
|      | $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ | .4, .3, .3 | .4, .4, .2 | .5, .3, .2 | .6, .2, .2 |
| 1.21 | 69571023                                | 69694619   | 70000484   | 70365789   | 71314574   |
| 1.22 | 69940324                                | 70061043   | 70356096   | 70711546   | 71640691   |
| 1.23 | 70305620                                | 70423462   | 70707774   | 71056355   | 71962931   |
| 1.24 | 70666944                                | 70781910   | 71055527   | 71397255   | 72281321   |
| 1.25 | 71024327                                | 71136421   | 71399431   | 71734275   | 72595914   |
| 1.26 | 71377800                                | 71487020   | 71739485   | 72067449   | 72906744   |
| 1.27 | 71727395                                | 71833758   | 72075741   | 72396762   | 73213874   |
| 1.28 | 72073144                                | 72176657   | 72408240   | 72722352   | 73517336   |
| 1.29 | 72415078                                | 72515739   | 72737005   | 73044247   | 73817166   |
| 1.30 | 72753230                                | 72851054   | 73062058   | 73362498   | 74113411   |
| 1.31 | 73087631                                | 73182620   | 73383457   | 73677080   | 74406123   |
| 1.32 | 73418313                                | 73510484   | 73701229   | 73987963   | 74695335   |
| 1.33 | 73745308                                | 73834667   | 74015418   | 74295283   | 74981088   |
| 1.34 | 74068648                                | 74155214   | 74326055   | 74599120   | 75263419   |
| 1.35 | 74388365                                | 74472143   | 74633155   | 74899504   | 75542378   |
| 1.36 | 74704491                                | 74785494   | 74936779   | 75196389   | 75818003   |
| 1.37 | 75017057                                | 75095296   | 75236940   | 75489757   | 76090334   |
| 1.38 | 75326096                                | 75401590   | 75533683   | 75779685   | 76359401   |
| 1.39 | 75631639                                | 75704400   | 75827034   | 76066306   | 76625256   |
| 1.40 | 75933717                                | 76003764   | 76117034   | 76349675   | 76887933   |
| 1.41 | 76232363                                | 76299706   | 76403723   | 76629769   | 77147469   |
| 1.42 | 76527608                                | 76592263   | 76687112   | 76906546   | 77403900   |
| 1.43 | 76819483                                | 76881469   | 76967249   | 77180080   | 77657268   |
| 1.44 | 77108020                                | 77167356   | 77244188   | 77450402   | 77907614   |
| 1.45 | 77393250                                | 77449958   | 77517922   | 77717642   | 78154958   |
| 1.46 | 77675204                                | 77729294   | 77788500   | 77981784   | 78399357   |
| 1.47 | 77953913                                | 78005408   | 78055954   | 78242762   | 78640840   |
| 1.48 | 78229408                                | 78278322   | 78320332   | 78500724   | 78879429   |
| 1.49 | 78501720                                | 78548084   | 78581647   | 78755611   | 79115182   |
| 1.50 | 78770879                                | 78814713   | 78839920   | 79007533   | 79348122   |
| 1.51 | 79036916                                | 79078232   | 79095202   | 79256549   | 79578279   |
| 1.52 | 79299862                                | 79338666   | 79347529   | 79502584   | 79805689   |
| 1.53 | 79559746                                | 79596081   | 79596924   | 79745751   | 80030400   |
| 1.54 | 79816599                                | 79850480   | 79843406   | 79986097   | 80252428   |
| 1.55 | 80070451                                | 80101902   | 80087025   | 80223569   | 80471816   |
| 1.56 | 80321332                                | 80350375   | 80327786   | 80458267   | 80688587   |
| 1.57 | 80569271                                | 80595926   | 80565752   | 80690151   | 80902779   |
| 1.58 | 80814298                                | 80838582   | 80800940   | 80919309   | 81114425   |
| 1.59 | 81056442                                | 81078395   | 81033363   | 81145831   | 81323569   |
| 1.60 | 81295732                                | 81315367   | 81263071   | 81369648   | 81530212   |

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

| t    | $a_3, a_2, a_1$ |            |            |            |              |
|------|-----------------|------------|------------|------------|--------------|
|      | .5, .4, .1      | .6, .3, .1 | .7, .2, .1 | .8, .1, .1 | .9, .05, .05 |
| 1.21 | 70533883        | 71172948   | 72253536   | 73108466   | 73008365     |
| 1.22 | 70858382        | 71485858   | 72540446   | 73355593   | 73221605     |
| 1.23 | 71179316        | 71795256   | 72823796   | 73599832   | 73432678     |
| 1.24 | 71496699        | 72101088   | 73103835   | 73841209   | 73641602     |
| 1.25 | 71810591        | 72403446   | 73380502   | 74079781   | 73848399     |
| 1.26 | 72121018        | 72702298   | 73653752   | 74315595   | 74053116     |
| 1.27 | 72427964        | 72997805   | 73923894   | 74548703   | 74255778     |
| 1.28 | 72731553        | 73289947   | 74190751   | 74779126   | 74456411     |
| 1.29 | 73031825        | 73578751   | 74454415   | 75006914   | 74655053     |
| 1.30 | 73328816        | 73864206   | 74715024   | 75232115   | 74851715     |
| 1.31 | 73622522        | 74146448   | 74972470   | 75454762   | 75046436     |
| 1.32 | 73912909        | 74425535   | 75227027   | 75674906   | 75239238     |
| 1.33 | 74200098        | 74701425   | 75478404   | 75892570   | 75430159     |
| 1.34 | 74484162        | 74974074   | 75726869   | 76107799   | 75619210     |
| 1.35 | 74765147        | 75243660   | 75972496   | 76320642   | 75806426     |
| 1.36 | 75042994        | 75510248   | 76215220   | 76531113   | 75991830     |
| 1.37 | 75317709        | 75773821   | 76455016   | 76739259   | 76175432     |
| 1.38 | 75589355        | 76034389   | 76692094   | 76945118   | 76357274     |
| 1.39 | 75858032        | 76291955   | 76926408   | 77148712   | 76537373     |
| 1.40 | 76123813        | 76546612   | 77157972   | 77350085   | 76715759     |
| 1.41 | 76386666        | 76798421   | 77386794   | 77549267   | 76892440     |
| 1.42 | 76646573        | 77047236   | 77613072   | 77746291   | 77067441     |
| 1.43 | 76903578        | 77293383   | 77836652   | 77941191   | 77240806     |
| 1.44 | 77157737        | 77536750   | 78057811   | 78133984   | 77412528     |
| 1.45 | 77409143        | 77777374   | 78276285   | 78324723   | 77582643     |
| 1.46 | 77657784        | 78015133   | 78492254   | 78513424   | 77751163     |
| 1.47 | 77903617        | 78250286   | 78705755   | 78700115   | 77918121     |
| 1.48 | 78146761        | 78482852   | 78916899   | 78884811   | 78083513     |
| 1.49 | 78387176        | 78712803   | 79125428   | 79067570   | 78247398     |
| 1.50 | 78624951        | 78940095   | 79331830   | 79248400   | 78409748     |
| 1.51 | 78860137        | 79164764   | 79535612   | 79427333   | 78570613     |
| 1.52 | 79092683        | 79387024   | 79737126   | 79604395   | 78730004     |
| 1.53 | 79322664        | 79606732   | 79936101   | 79779622   | 78887941     |
| 1.54 | 79550134        | 79824012   | 80132979   | 79953018   | 79044435     |
| 1.55 | 79775066        | 80038683   | 80327784   | 80124624   | 79199512     |
| 1.56 | 79997519        | 80251120   | 80520591   | 80294447   | 79353183     |
| 1.57 | 80217471        | 80461058   | 80711134   | 80462526   | 79505466     |
| 1.58 | 80435003        | 80668810   | 80899667   | 80628891   | 79656378     |
| 1.59 | 80650191        | 80873984   | 81086043   | 80793544   | 79805945     |
| 1.60 | 80862971        | 81077100   | 81270026   | 80956529   | 79954163     |

Table 2

$$P[a_1^{(1)}x_1^2 + a_2^{(2)}x_1^2 + a_3^{(3)}x_1^2 \leq t]$$

| t    | $a_3, a_2, a_1$                         |            |            |            |            |
|------|---|------------|------------|------------|------------|
|      | $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ | .4, .3, .3 | .4, .4, .2 | .5, .3, .2 | .6, .2, .2 |
| 1.61 | 81532197                                | 81549533   | 81490083   | 81590833   | 81734412   |
| 1.62 | 81765867                                | 81780928   | 81714438   | 81809324   | 81936185   |
| 1.63 | 81996769                                | 82009582   | 81936156   | 82025241   | 82135558   |
| 1.64 | 82224932                                | 82235519   | 82155266   | 82238664   | 82332576   |
| 1.65 | 82450385                                | 82458784   | 82371806   | 82449578   | 82527263   |
| 1.66 | 82673155                                | 82679370   | 82585777   | 82657984   | 82719643   |
| 1.67 | 82893271                                | 82897341   | 82797227   | 82863851   | 82909733   |
| 1.68 | 83110760                                | 83112699   | 83006193   | 83067684   | 83097592   |
| 1.69 | 83325649                                | 83325494   | 83212692   | 83269969   | 83283226   |
| 1.70 | 83537966                                | 83535727   | 83416759   | 83468917   | 83466668   |
| 1.71 | 83747738                                | 83743446   | 83618386   | 83663252   | 83647943   |
| 1.72 | 83954992                                | 83948673   | 83817649   | 83857239   | 83827083   |
| 1.73 | 84159755                                | 84151435   | 84014540   | 84048823   | 84004099   |
| 1.74 | 84362054                                | 84351754   | 84209109   | 84238337   | 84179051   |
| 1.75 | 84561914                                | 84549668   | 84401355   | 84426945   | 84351932   |
| 1.76 | 84759362                                | 84745182   | 84591329   | 84610263   | 84522779   |
| 1.77 | 84954424                                | 84938346   | 84779033   | 84792875   | 84691617   |
| 1.78 | 85147125                                | 85179178   | 84964514   | 84973296   | 84858480   |
| 1.79 | 85337491                                | 85317688   | 85147791   | 85151703   | 85023379   |
| 1.80 | 85525548                                | 85503917   | 85328895   | 85327976   | 85186343   |
| 1.81 | 85711320                                | 85687888   | 85507814   | 85502090   | 85347398   |
| 1.82 | 85894833                                | 85869617   | 85684620   | 85674210   | 85506566   |
| 1.83 | 86076111                                | 86049149   | 85859313   | 85843638   | 85663867   |
| 1.84 | 86255179                                | 86226502   | 86031918   | 86014098   | 85819334   |
| 1.85 | 86432061                                | 86401676   | 86202455   | 86179646   | 85972998   |
| 1.86 | 86606781                                | 86574728   | 86370956   | 86342005   | 86124847   |
| 1.87 | 86779363                                | 86745662   | 86537443   | 86504190   | 86274930   |
| 1.88 | 86949831                                | 86914503   | 86701948   | 86664091   | 86423261   |
| 1.89 | 87118208                                | 87081273   | 86864471   | 86822260   | 86569879   |
| 1.90 | 87284517                                | 87246018   | 87025062   | 86978592   | 86714776   |
| 1.91 | 87448782                                | 87408738   | 87183715   | 87132947   | 86857994   |
| 1.92 | 87611026                                | 87569446   | 87340462   | 87285703   | 86999546   |
| 1.93 | 87771271                                | 87728203   | 87495326   | 87436231   | 87139464   |
| 1.94 | 87929540                                | 87884985   | 87648347   | 87585143   | 87277743   |
| 1.95 | 88085855                                | 88039860   | 87799521   | 87732358   | 87414431   |
| 1.96 | 88240238                                | 88192813   | 87948877   | 87877623   | 87549536   |
| 1.97 | 88392711                                | 88343890   | 88096426   | 88021430   | 87683077   |
| 1.98 | 88543296                                | 88493097   | 88242220   | 88163168   | 87815073   |
| 1.99 | 88692014                                | 88640471   | 88386242   | 88303850   | 87945539   |
| 2.00 | 88838886                                | 88786014   | 88528546   | 88443449   | 88074518   |

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

| t    | $a_3, a_2, a_1$ |            |            |            |              |
|------|-----------------|------------|------------|------------|--------------|
|      | .5, .4, .1      | .6, .3, .1 | .7, .2, .1 | .8, .1, .1 | .9, .05, .05 |
| 1.61 | 81073402        | 81277787   | 81451888   | 81117860   | 80101069     |
| 1.62 | 81281451        | 81476331   | 81631960   | 81277553   | 80246662     |
| 1.63 | 81487204        | 81672555   | 81810163   | 81435630   | 80390965     |
| 1.64 | 81690733        | 81866634   | 81986641   | 81592115   | 80533993     |
| 1.65 | 81892026        | 82058548   | 82160807   | 81747043   | 80675772     |
| 1.66 | 82091087        | 82248281   | 82332852   | 81900407   | 80816288     |
| 1.67 | 82287891        | 82435974   | 82502978   | 82052244   | 80955572     |
| 1.68 | 82482476        | 82621446   | 82671175   | 82202570   | 81093651     |
| 1.69 | 82674960        | 82804954   | 82837616   | 82351404   | 81230517     |
| 1.70 | 82865372        | 82986353   | 83002509   | 82498769   | 81366200     |
| 1.71 | 83053673        | 83165869   | 83165788   | 82644687   | 81500706     |
| 1.72 | 83239883        | 83343203   | 83327490   | 82789158   | 81634042     |
| 1.73 | 83424004        | 83518624   | 83487218   | 82932213   | 81766231     |
| 1.74 | 83606069        | 83692153   | 83644868   | 83073879   | 81897291     |
| 1.75 | 83786146        | 83863723   | 83800576   | 83214150   | 82027220     |
| 1.76 | 83964248        | 84033475   | 83954681   | 83353051   | 82156033     |
| 1.77 | 84140324        | 84201119   | 84107192   | 83490619   | 82283757     |
| 1.78 | 84314427        | 84367084   | 84258325   | 83626847   | 82410391     |
| 1.79 | 84486725        | 84531141   | 84407850   | 83761764   | 82535938     |
| 1.80 | 84657100        | 84693473   | 84555904   | 83895373   | 82660438     |
| 1.81 | 84825558        | 84853915   | 84702224   | 84027703   | 82783870     |
| 1.82 | 84992225        | 85012605   | 84846703   | 84158766   | 82906275     |
| 1.83 | 85156821        | 85169525   | 84989189   | 84288571   | 83027648     |
| 1.84 | 85319805        | 85324735   | 85130352   | 84417142   | 83147998     |
| 1.85 | 85480977        | 85478216   | 85270146   | 84544499   | 83267353     |
| 1.86 | 85640331        | 85630073   | 85408664   | 84670634   | 83385701     |
| 1.87 | 85798071        | 85780142   | 85545903   | 84795575   | 83503067     |
| 1.88 | 85953772        | 85928554   | 85681580   | 84919331   | 83619455     |
| 1.89 | 86107911        | 86075426   | 85815658   | 85041921   | 83734874     |
| 1.90 | 86260387        | 86220659   | 85948316   | 85163370   | 83849352     |
| 1.91 | 86411088        | 86364219   | 86079479   | 85283674   | 83962879     |
| 1.92 | 86560340        | 86506365   | 86209492   | 85402845   | 84075466     |
| 1.93 | 86707591        | 86646781   | 86338175   | 85520917   | 84187139     |
| 1.94 | 86853388        | 86785748   | 86465742   | 85637874   | 84297877     |
| 1.95 | 86997647        | 86923169   | 86592052   | 85753748   | 84407732     |
| 1.96 | 87140152        | 87059041   | 86717102   | 85868546   | 84516676     |
| 1.97 | 87281325        | 87193479   | 86840790   | 85982284   | 84624747     |
| 1.98 | 87420765        | 87326444   | 86962614   | 86094967   | 84731925     |
| 1.99 | 87558612        | 87457842   | 87082913   | 86206610   | 84838250     |
| 2.00 | 87695129        | 87588007   | 87202116   | 86317224   | 84943710     |



Table 2

$$P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 + a_3^{(3)}\chi_1^2 \leq t]$$

| t    | $a_3, a_2, a_1$                         |            |            |            |            |
|------|---|------------|------------|------------|------------|
|      | $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ | .4, .3, .3 | .4, .4, .2 | .5, .3, .2 | .6, .2, .2 |
| 2.05 | 89546295                                | 89487135   | 89214681   | 89110327   | 88697394   |
| 2.10 | 90210643                                | 90145786   | 89860380   | 89739430   | 89285361   |
| 2.15 | 90834371                                | 90764360   | 90467927   | 90331769   | 89840482   |
| 2.20 | 91419800                                | 91345190   | 91039533   | 90889539   | 90364747   |
| 2.25 | 91969136                                | 91890458   | 91577261   | 91414828   | 90859986   |
| 2.30 | 92484474                                | 92402208   | 92083048   | 91908707   | 91327957   |
| 2.35 | 92967802                                | 92882431   | 92558763   | 92373968   | 91770256   |
| 2.40 | 93421003                                | 93332957   | 93006156   | 92812182   | 92188405   |
| 2.45 | 93845861                                | 93755588   | 93426875   | 93224614   | 92583837   |
| 2.50 | 94244063                                | 94151949   | 93822474   | 93613050   | 92957875   |
| 2.55 | 94617206                                | 94523627   | 94194441   | 93977778   | 93311771   |
| 2.60 | 94966798                                | 94872101   | 94544162   | 94321818   | 93646700   |
| 2.65 | 95294264                                | 95198782   | 94872954   | 94646642   | 93963753   |
| 2.70 | 95600949                                | 95505000   | 95182042   | 94951306   | 94263964   |
| 2.75 | 95888121                                | 95791965   | 95472596   | 95239517   | 94548283   |
| 2.80 | 96156976                                | 96060882   | 95745714   | 95509422   | 94817634   |
| 2.85 | 96408642                                | 96312841   | 96002432   | 95764748   | 95072858   |
| 2.90 | 96644181                                | 96548899   | 96243723   | 96005972   | 95314747   |
| 2.95 | 96864593                                | 96770036   | 96470511   | 96231416   | 95544065   |
| 3.00 | 97070819                                | 96977158   | 96683656   | 96445409   | 95761501   |
| 3.05 | 97263745                                | 97171137   | 96883975   | 96645543   | 95967718   |
| 3.10 | 97444205                                | 97352811   | 97072230   | 96833972   | 96163346   |
| 3.15 | 97612981                                | 97522933   | 97249151   | 97012452   | 96348947   |
| 3.20 | 97770810                                | 97682231   | 97415415   | 97180218   | 96525098   |
| 3.25 | 97918383                                | 97831356   | 97571641   | 97339631   | 96692286   |
| 3.30 | 98056350                                | 97970971   | 97718459   | 97490258   | 96851029   |
| 3.35 | 98185322                                | 98101658   | 97856418   | 97629034   | 97001756   |
| 3.40 | 98305871                                | 98224009   | 97986047   | 97761128   | 97144911   |
| 3.45 | 98418535                                | 98338519   | 98107864   | 97886202   | 97280905   |
| 3.50 | 98523819                                | 98445692   | 98222320   | 98005301   | 97410088   |
| 3.55 | 98622195                                | 98545992   | 98329858   | 98116926   | 97532880   |
| 3.60 | 98714108                                | 98639871   | 98430910   | 98220848   | 97649560   |
| 3.65 | 98799974                                | 98727721   | 98525846   | 98318904   | 97760466   |
| 3.70 | 98880183                                | 98809895   | 98615053   | 98411043   | 97865924   |
| 3.75 | 98955101                                | 98886818   | 98698870   | 98498553   | 97966192   |
| 3.80 | 99025071                                | 98958777   | 98777615   | 98581928   | 98061552   |
| 3.85 | 99090414                                | 99026104   | 98851600   | 98661118   | 98152250   |
| 3.90 | 99151430                                | 99089087   | 98921121   | 98734631   | 98238527   |
| 3.95 | 99208401                                | 99148025   | 98986428   | 98804327   | 98320624   |
| 4.00 | 99261591                                | 99203157   | 99047792   | 98870820   | 98398744   |

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

| t    | $a_3, a_2, a_1$ |            |            |            |              |
|------|-----------------|------------|------------|------------|--------------|
|      | .5, .4, .1      | .6, .3, .1 | .7, .2, .1 | .8, .1, .1 | .9, .05, .05 |
| 2.05 | 88354913        | 88216198   | 87780291   | 86855286   | 85458391     |
| 2.10 | 88979083        | 88812074   | 88331407   | 87369325   | 85952851     |
| 2.15 | 89569620        | 89375782   | 88853948   | 87860617   | 86428052     |
| 2.20 | 90128405        | 89910515   | 89349301   | 88330367   | 86884938     |
| 2.25 | 90656908        | 90415299   | 89820246   | 88779680   | 87324360     |
| 2.30 | 91157167        | 90895837   | 90270479   | 89209628   | 87747125     |
| 2.35 | 91629964        | 91349222   | 90697422   | 89621181   | 88154018     |
| 2.40 | 92077680        | 91780213   | 91104658   | 90015238   | 88545741     |
| 2.45 | 92501146        | 92186836   | 91491567   | 90392689   | 88922989     |
| 2.50 | 92902054        | 92573273   | 91860421   | 90754341   | 89286381     |
| 2.55 | 93279965        | 92939208   | 92211597   | 91100944   | 89636537     |
| 2.60 | 93638484        | 93287970   | 92547482   | 91433226   | 89974018     |
| 2.65 | 93978470        | 93617955   | 92866833   | 91751864   | 90299375     |
| 2.70 | 94298811        | 93929089   | 93171374   | 92057508   | 90613117     |
| 2.75 | 94603532        | 94225421   | 93461863   | 92350734   | 90915709     |
| 2.80 | 94889999        | 94508391   | 93739531   | 92632141   | 91207629     |
| 2.85 | 95162411        | 94776018   | 94003351   | 92902251   | 91489315     |
| 2.90 | 95421082        | 95027702   | 94255156   | 93161583   | 91761175     |
| 2.95 | 95663636        | 95266775   | 94496070   | 93410613   | 92023605     |
| 3.00 | 95895262        | 95494309   | 94726120   | 93649811   | 92276975     |
| 3.05 | 96113132        | 95712151   | 94950300   | 93879609   | 92521639     |
| 3.10 | 96317749        | 95918844   | 95160142   | 94100412   | 92757943     |
| 3.15 | 96513305        | 96114154   | 95362175   | 94312606   | 92986207     |
| 3.20 | 96697909        | 96301144   | 95554224   | 94516588   | 93206739     |
| 3.25 | 96874467        | 96477685   | 95737147   | 94712687   | 93419843     |
| 3.30 | 97041236        | 96645137   | 95914696   | 94901252   | 93625782     |
| 3.35 | 97196091        | 96804607   | 96081396   | 95082605   | 93824830     |
| 3.40 | 97343808        | 96956103   | 96241050   | 95257053   | 94017259     |
| 3.45 | 97484354        | 97101154   | 96393815   | 95424871   | 94203298     |
| 3.50 | 97619099        | 97238859   | 96540323   | 95586345   | 94383185     |
| 3.55 | 97745707        | 97369230   | 96680574   | 95741735   | 94557134     |
| 3.60 | 97863181        | 97493489   | 96815896   | 95891293   | 94725383     |
| 3.65 | 97974514        | 97611594   | 96944015   | 96035260   | 94888124     |
| 3.70 | 98079363        | 97723852   | 97066910   | 96173869   | 95045559     |
| 3.75 | 98179666        | 97830966   | 97184781   | 96307325   | 95197866     |
| 3.80 | 98275968        | 97931832   | 97297600   | 96435847   | 95345247     |
| 3.85 | 98368061        | 98027868   | 97405544   | 96559618   | 95487846     |
| 3.90 | 98453351        | 98119631   | 97508867   | 96678852   | 95625857     |
| 3.95 | 98534810        | 98207384   | 97608002   | 96793708   | 95759446     |
| 4.00 | 98613136        | 98290648   | 97703106   | 96904369   | 95888737     |

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

 $a_3, a_2, a_1$ 

| t     | $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ | .4, .3, .3 | .4, .4, .2 | .5, .3, .2 | .6, .2, .2 |
|-------|---|------------|------------|------------|------------|
| 4.05  | 99311248                                | 99254732   | 99105433   | 98932982   | 98473077   |
| 4.10  | 99357602                                | 99302982   | 99159601   | 98991239   | 98543841   |
| 4.15  | 99400869                                | 99348104   | 99210483   | 99046538   | 98611203   |
| 4.20  | 99441253                                | 99390313   | 99258288   | 99098587   | 98675338   |
| 4.25  | 99478942                                | 99429791   | 99303197   | 99147863   | 98736382   |
| 4.30  | 99514114                                | 99466723   | 99345394   | 99194833   | 98794540   |
| 4.35  | 99546935                                | 99501260   | 99385042   | 99239175   | 98849921   |
| 4.40  | 99577560                                | 99533570   | 99422282   | 99280954   | 98902675   |
| 4.45  | 99606134                                | 99563793   | 99457265   | 99320027   | 98952918   |
| 4.50  | 99632793                                | 99592049   | 99490132   | 99357214   | 99000788   |
| 4.55  | 99657663                                | 99618472   | 99521016   | 99392335   | 99046401   |
| 4.60  | 99680864                                | 99643194   | 99550021   | 99425331   | 99089876   |
| 4.65  | 99702505                                | 99666317   | 99577279   | 99456826   | 99131309   |
| 4.70  | 99722691                                | 99687942   | 99602881   | 99486196   | 99170783   |
| 4.75  | 99741518                                | 99708164   | 99626935   | 99513776   | 99208411   |
| 4.80  | 99759077                                | 99727063   | 99649530   | 99539851   | 99244281   |
| 4.85  | 99775452                                | 99744759   | 99670762   | 99564584   | 99278494   |
| 4.90  | 99790723                                | 99761298   | 99690705   | 99588132   | 99311099   |
| 4.95  | 99804963                                | 99776739   | 99709438   | 99610412   | 99342184   |
| 5.00  | 99818241                                | 99791215   | 99727037   | 99631387   | 99371845   |
| 5.50  | 99910366                                | 99893092   | 99853888   | 99792144   | 99602675   |
| 6.00  | 99955921                                | 99945210   | 99921776   | 99879548   | 99747362   |
| 6.50  | 99978361                                | 99971879   | 99958121   | 99929510   | 99838653   |
| 7.00  | 99989379                                | 99985542   | 99977534   | 99958993   | 99896550   |
| 7.50  | 99994774                                | 99992557   | 99987873   | 99975736   | 99933430   |
| 8.00  | 99997409                                | 99996155   | 99993285   | 99985744   | 99957027   |
| 8.50  | 99998693                                | 99998012   | 99995956   | 99991490   | 99972177   |
| 9.00  | 99999318                                | 99998970   | 99997001   | 99994914   | 99981946   |
| 9.50  | 99999622                                | 99999469   | 99996967   | 99996929   | 99988256   |
| 10.00 | 99999769                                | 99999727   | 99996025   | 99998157   | 99992336   |

Table 2

$$P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 + a_3^{(3)}\chi_1^2 \leq t]$$

| t     | $a_3, a_2, a_1$ |            |            |            |              |
|-------|-----------------|------------|------------|------------|--------------|
|       | .5, .4, .1      | .6, .3, .1 | .7, .2, .1 | .8, .1, .1 | .9, .05, .05 |
| 4.05  | 98686317        | 98380785   | 97796029   | 97011003   | 96013892     |
| 4.10  | 98754587        | 98455644   | 97883325   | 97113763   | 96135063     |
| 4.15  | 98819888        | 98526963   | 97966790   | 97212803   | 96252375     |
| 4.20  | 98881368        | 98598747   | 98046741   | 97308282   | 96365963     |
| 4.25  | 98939814        | 98663605   | 98123396   | 97400308   | 96475953     |
| 4.30  | 98995976        | 98725104   | 98196950   | 97489031   | 96582473     |
| 4.35  | 99049364        | 98783236   | 98267583   | 97574576   | 96685638     |
| 4.40  | 99099893        | 98845291   | 98335372   | 97657088   | 96785551     |
| 4.45  | 99146907        | 98897419   | 98400380   | 97736638   | 96882339     |
| 4.50  | 99192140        | 98947306   | 98462847   | 97813369   | 96976071     |
| 4.55  | 99234795        | 98994798   | 98522832   | 97887382   | 97066896     |
| 4.60  | 99274655        | 99040200   | 98580422   | 97958773   | 97154874     |
| 4.65  | 99312916        | 99083668   | 98635671   | 98027656   | 97240118     |
| 4.70  | 99348880        | 99125279   | 98689384   | 98094109   | 97322715     |
| 4.75  | 99382467        | 99164810   | 98740073   | 98158224   | 97402747     |
| 4.80  | 99414315        | 99204974   | 98788730   | 98220101   | 97480299     |
| 4.85  | 99444725        | 99240521   | 98835411   | 98279808   | 97555447     |
| 4.90  | 99473941        | 99274531   | 98880262   | 98337449   | 97628289     |
| 4.95  | 99501818        | 99307065   | 98923355   | 98393061   | 97698875     |
| 5.00  | 99527881        | 99338204   | 98964765   | 98446749   | 97767299     |
| 5.50  | 99750905        | 99583105   | 99298686   | 98891694   | 98346329     |
| 6.00  | 99856212        | 99733650   | 99523265   | 99206295   | 98771996     |
| 6.50  | 99914625        | 99830209   | 99674216   | 99429879   | 99086089     |
| 7.00  | 99951798        | 99890877   | 99776655   | 99589460   | 99318510     |
| 7.50  | 99971116        | 99929617   | 99846479   | 99703766   | 99490992     |
| 8.00  | 99983293        | 99954935   | 99894153   | 99785854   | 99619258     |
| 8.50  | 99989933        | 99970724   | 99926946   | 99844959   | 99714823     |
| 9.00  | 99993949        | 99980952   | 99949396   | 99887597   | 99786149     |
| 9.50  | 99996324        | 99987561   | 99964908   | 99918425   | 99839456     |
| 10.00 | 99997911        | 99991860   | 99975591   | 99940714   | 99879378     |

Table 3  
 $P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 \leq t]$

| $a_2, a_1$ |           |           |                            |           |           |
|------------|-----------|-----------|----------------------------|-----------|-----------|
| P          | .5, .5    | .6, .4    | $\frac{2}{3}, \frac{1}{3}$ | .7, .3    | .75, .25  |
| .05        | .0512933  | .0502834  | .0484347                   | .0471210  | .0445980  |
| .10        | .1053605  | .1033439  | .0996528                   | .0970299  | .0919943  |
| .15        | .1625189  | .1595026  | .1539828                   | .1500613  | .1425363  |
| .20        | .2231436  | .2191393  | .2118145                   | .2066130  | .1966385  |
| .25        | .2876821  | .2827083  | .2736149                   | .2671620  | .2548001  |
| .30        | .3566749  | .3507583  | .3399502                   | .3322877  | .3176293  |
| .35        | .4307829  | .4239617  | .4115148                   | .4027026  | .3858766  |
| .40        | .5108256  | .5031533  | .4891744                   | .4792956  | .4604821  |
| .45        | .5978370  | .5893882  | .5740254                   | .5631956  | .5426442  |
| .50        | .6931472  | .6840256  | .6674860                   | .6558649  | .6339206  |
| .55        | .7985077  | .7888589  | .7714293                   | .7592430  | .7363857  |
| .60        | .9162907  | .9063221  | .8884132                   | .8759775  | .8528807  |
| .65        | 1.0498221 | 1.0398349 | 1.0220402                  | 1.0098117 | .9874373  |
| .70        | 1.2039728 | 1.1944190 | 1.1776240                  | 1.1662788 | 1.1460333 |
| .75        | 1.3862944 | 1.3778818 | 1.3634634                  | 1.3540430 | 1.3380548 |
| .80        | 1.6094379 | 1.6033472 | 1.5935807                  | 1.5877814 | 1.5794361 |
| .85        | 1.8971200 | 1.8955189 | 1.8945236                  | 1.8953551 | 1.9004575 |
| .90        | 2.3025851 | 2.3101499 | 2.3266188                  | 2.3402352 | 2.3700817 |
| .95        | 2.9957323 | 3.0265143 | 3.0855245                  | 3.1287252 | 3.2119173 |

Table 3

$$P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 \leq t]$$

| $a_2, a_1$ |            |           |           |           |           |
|------------|------------|-----------|-----------|-----------|-----------|
| .8, .2     | .875, .125 | .9, .1    | .95, .05  | .99, .01  | 1, 0      |
| .0412995   | 0.0343785  | 0.0313116 | 0.0231255 | 0.0113775 | 0.0039321 |
| .0854130   | 0.0716190  | 0.0655149 | 0.0492758 | 0.0264839 | 0.0157908 |
| .1327088   | 0.1121513  | 0.1030800 | 0.0791007 | 0.0469824 | 0.0357658 |
| .1836280   | 0.1564972  | 0.1445808 | 0.1134158 | 0.0747420 | 0.0641848 |
| .2387046   | 0.2052967  | 0.1907259 | 0.1532382 | 0.1112559 | 0.1015311 |
|            |            |           |           |           |           |
| .2985925   | 0.2593408  | 0.2423946 | 0.1998188 | 0.1574586 | 0.1484719 |
| .3641019   | 0.3196192  | 0.3006879 | 0.2546648 | 0.2141737 | 0.2059002 |
| .4362531   | 0.3873841  | 0.3669949 | 0.3195510 | 0.2824997 | 0.2749959 |
| .5163519   | 0.4642405  | 0.4430833 | 0.3965315 | 0.3639473 | 0.3573172 |
| .6061019   | 0.5522766  | 0.5312252 | 0.4879979 | 0.4605556 | 0.4549364 |
|            |            |           |           |           |           |
| .7077789   | 0.6542579  | 0.6343803 | 0.5968432 | 0.5750892 | 0.5706519 |
| .8245074   | 0.7739286  | 0.7564806 | 0.7268075 | 0.7113684 | 0.7083264 |
| .9607254   | 0.9165067  | 0.9029067 | 0.8830853 | 0.8748335 | 0.8734573 |
| 1.1230118  | 1.0895570  | 1.0813581 | 1.0733542 | 1.0735506 | 1.0741943 |
| 1.3216774  | 1.3046746  | 1.3035784 | 1.3096868 | 1.3201595 | 1.3233038 |
|            |            |           |           |           |           |
| 1.5741595  | 1.5811170  | 1.5891177 | 1.6125527 | 1.6360350 | 1.6423745 |
| 1.9134105  | 1.9548644  | 1.9747334 | 2.0207290 | 2.0616020 | 2.0722508 |
| 2.4139603  | 2.5070381  | 2.5436905 | 2.6221806 | 2.6885579 | 2.7055437 |
| 3.3157800  | 3.4993314  | 3.5651446 | 3.7011350 | 3.8131053 | 3.8414598 |

Table 4

$$P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 + a_3^{(3)}\chi_1^2 \leq t]$$

| $a_3, a_2, a_1$ |   |            |            |            |            |
|-----------------|---|------------|------------|------------|------------|
| P               | $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ | .4, .3, .3 | .4, .4, .2 | .5, .3, .2 | .6, .2, .2 |
| .05             | .1172838                                | .1162503   | .1121598   | .1098702   | .1023767   |
| .10             | .1947929                                | .1931584   | .1867800   | .1830821   | .1710399   |
| .15             | .2659252                                | .2637962   | .2556041   | .2506954   | .2347879   |
| .20             | .3350594                                | .3325023   | .3228059   | .3168023   | .2974412   |
| .25             | .4041790                                | .4012462   | .3902967   | .3832849   | .3607890   |
| .30             | .4745521                                | .4712908   | .4593189   | .4513756   | .4260352   |
| .35             | .5471933                                | .5436504   | .5308865   | .5220894   | .4942012   |
| .40             | .6230577                                | .6192834   | .6059725   | .5964080   | .5663042   |
| .45             | .7031571                                | .6992088   | .6856241   | .6753956   | .6434740   |
| .50             | .7886597                                | .7846056   | .7710647   | .7603039   | .7270653   |
| .55             | .8810036                                | .8769286   | .8638143   | .8526958   | .8187967   |
| .60             | .9820574                                | .9780719   | .9658609   | .9546252   | .9209578   |
| .65             | 1.0943731                               | 1.0906255  | 1.0799350  | 1.0689260  | 1.0367462  |
| .70             | 1.2216261                               | 1.2183264  | 1.2099865  | 1.1997185  | 1.1708636  |
| .75             | 1.3694513                               | 1.3669116  | 1.3620965  | 1.3533718  | 1.3306658  |
| .80             | 1.5472127                               | 1.5459324  | 1.5464142  | 1.5405821  | 1.5286400  |
| .85             | 1.7723537                               | 1.7732174  | 1.7819266  | 1.7814896  | 1.7885747  |
| .90             | 2.0838033                               | 2.0886598  | 2.1112237  | 2.1216246  | 2.1649136  |
| .95             | 2.6049232                               | 2.6191934  | 2.6701791  | 2.7082268  | 2.8354503  |

Table 4

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

| $a_3, a_2, a_1$ |            |            |            |            |              |
|-----------------|------------|------------|------------|------------|--------------|
| P               | .5, .4, .1 | .6, .3, .1 | .7, .2, .1 | .8, .1, .1 | .9, .05, .05 |
| .05             | .0976010   | .0942377   | .0867553   | .0725375   | .0489181     |
| .10             | .1643891   | .1587611   | .1463208   | .1230581   | .0848072     |
| .15             | .2272579   | .2195558   | .2026281   | .1714381   | .1207631     |
| .20             | .2897753   | .2800898   | .2589169   | .2204765   | .1589431     |
| .25             | .3536368   | .3420285   | .3167887   | .2716613   | .2007928     |
| .30             | .4200059   | .4065320   | .3773992   | .3261687   | .2477116     |
| .35             | .4898878   | .4746180   | .4418026   | .3851624   | .3012680     |
| .40             | .5642940   | .5473257   | .5111112   | .4499504   | .3633136     |
| .45             | .6443518   | .6258263   | .5866130   | .5221161   | .4360567     |
| .50             | .7314079   | .7115335   | .6698963   | .6036681   | .5221283     |
| .55             | .8271599   | .8062430   | .7630155   | .6972421   | .6246956     |
| .60             | .9338467   | .9123419   | .8687372   | .8063995   | .7477316     |
| .65             | 1.0545596  | 1.0331489  | .9909504   | .9341040   | .8965913     |
| .70             | 1.1937848  | 1.1735081  | 1.1354033  | 1.0935425  | 1.0791032    |
| .75             | 1.3584452  | 1.3409569  | 1.3110758  | 1.2896947  | 1.3076064    |
| .80             | 1.5601119  | 1.5481910  | 1.5332340  | 1.5427275  | 1.6031101    |
| .85             | 1.8204696  | 1.8192014  | 1.8307625  | 1.8865691  | 2.0053720    |
| .90             | 2.1882602  | 2.2086801  | 2.2696650  | 2.3980261  | 2.6039266    |
| .95             | 2.8199090  | 2.8943630  | 3.0615976  | 3.3269842  | 3.6853594    |



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